CREOL OSE 6120: Theoretical Foundations of Optics
College of Optics and Photonics, Fall 2017
University of Central Florida

COURSE SYLLABUS

Instructor:	Dr. Boris Zeldovich	Term: Fall 2017
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TA: None

I. Welcome!

Welcome to CREOL OSE 6120: Theoretical Foundations of Optics.

II. University Course Catalog Description:

The focus of the course is to understand mathematical physics of optical processes, as described by ordinary and partial differential equations, wave propagation, Fourier transformations.

III. Course Overview:

The topics to be covered in this course are: Elements of Linear Algebra. Orthogonal Expansions. Fourier series (in narrow sense), Fourier integrals. Numerous eigenfunctions of Mathematical Fourier transform operator, including Hermit-Gaussian modes / quantum oscillator wave functions. Hyperbolic secant as an eigenfunction of that operator. Ordinary Differential Equations (ODEs): linear, nonlinear; systems of ODE. Cauchy problem. Problem with boundary conditions at different points. Partial Differential Equations (PDEs). Specific role of impedance General solution with the use of Green’s function: exact and approximate Huygens’ principle. Analytic Functions of Complex Variable. Special functions (Airy, Bessel, and other) as solutions of ODEs. “Stitching” and choice out of two linearly independent solutions in different regions of argument. Necessary topic (probably at the beginning of the course): Newton’s binomial formula for arbitrary (positive/negative, integer/fractional, real/complex) values of power.

IV. Course Objectives:

- To learn fast solution of typical ODEs.
- To understand superposition principle and eigenvectors of Hermitian operator.
- To understand how to calculate functions of matrices via Lagrange interpolation formula.
- To understand asymptotic solutions of wave equation. Course Prerequisites:

Graduate Standing, or C.I.

V. Course Credits:

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VI. Text Book:

G. J. Gbur, Mathematical Methods for Optical Physics and Engineering (Cambridge University, 2011).

VII. Reference Book:

M. Abramovitz and I. A. Stegun, Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables.

VIII. Basis for Final Grade:

Assessment	Percent of Final Grade
Homework	40%
Mid-term exam	20%
Final Exam	40%
	100%

Grading scale:

Grading Scale (%)	
90-100	A
80 - 89	B
70 - 79	C
60 - 69	D
0 - 59	F

IX. Grade Dissemination

Graded tests and materials in this course will be returned individually only by request. You can access your scores at any time using "myUCF Grades" in the portal. Please note that scores returned mid-semester are unofficial grades.

X. Course Policies: Grades

Late Work Policy:

There are **no** make-ups for the homework, or mid-term, or the final exam. Homework will be assessed a penalty: 10 points will be deduced if it is one day late, and will not be accepted if overdue by more than seven days.

Extra Credit Policy: No extra credit will be offered.

Grades of "Incomplete":

The current university policy concerning incomplete grades will be followed in this course. Incomplete grades are given only in situations where *unexpected emergencies prevent a student from completing the course and the remaining work can be completed the next semester*. Instructor is the final authority on whether you qualify for an incomplete. Incomplete work must be finished by the end of the subsequent semester or the "I" will automatically be recorded as an "F" on your transcript.

Grade Posting: Grades will be given to student individually and not posted publicly.

XI. Course Policies: Technology and Media

Email: Please use email for all important correspondence.

XII. Course Policies: Student Expectations

Disability Access: The University of Central Florida is committed to providing reasonable accommodations for all persons with disabilities. Students with disabilities who need accommodations in this course must contact the professor at the beginning of the semester to discuss needed accommodations. No accommodations will be provided until the student has met with the professor to request accommodations. Students who need accommodations must be registered with Student Disability Services, Student Resource Center Room 132, phone (407) 823-2371, TTY/TDD only phone (407) 823-2116, before requesting accommodations from the professor.

Attendance Policy:

- Regular class attendance is strongly advised and is necessary for students to understand many of the topics covered.
- Students must be on time to class.
- If missed a class, it is the responsibility of the student to find out the materials covered.

XIII. Financial Aid Statement

Financial Aid and Attendance: As of Fall 2014, all faculty members are required to document students' academic activity at the beginning of each course. In order to document that you began this course, please complete the following academic activity by the end of the first week of classes, or as soon as possible after adding the course, but no later than August 27. Failure to do so will result in a delay in the disbursement of your financial aid.

Details

0. Fundamental theorems of Arithmetic, of Algebra

(without rigorous proofs, but with counter-examples. Notions of Ring, Field, Group. Fields F_p.)

1. Elements of Linear Algebra.

The purpose of this Section 1 is the wide generalization of Superposition Principle. To be covered:
1.1. Linear Space over the fields of a) real numbers, b) complex numbers as an expression of un-sensitivity to a phase shift, c) shortly – over other fields of numbers.

1.2 Linear and bi-linear forms. Definition of Scalar (Dot) Product in real and complex spaces; definition of Euclidean Space as a linear space with a scalar product. Only after that, definition of Symmetric and Hermitian Operators/matrices in real and complex Euclidean spaces, respectively. Definition of Orthogonal and Unitary Operators. Connection with conservation of Energy or Probability.

1.3. Eigenvalues and eigenvectors of a linear operator. Possibility of reduction of a Symmetric / Hermitian operator to diagonal form via rotation of coordinate system. Completeness of the system of eigenvectors of a general operator: when it takes place and when it possibly does not. “Additional” Voigt’s waves.

1.4. Matrices, powers of matrices and functions of matrices. Thesis: “Functions of matrices are not so much different from the functions of one variable, x or $z = x + iy$”. Characteristic polynomial and characteristic equation for eigenvalues; **Cayley–Hamilton’s theorem. Solving Coupled Wave Equations via “Interpolation Lagrange Polynomial”.**

1.5. Use of software to handle matrices.

2. Orthogonal Expansions.

The purpose of this section is to familiarize students with particular systems of functions used for the representation of solutions of various physical problems.
2.1. Different possibilities to define scalar product in the linear space of functions. Gramm-Schmidt orthogonalization of polynomials. Study of particular “orthogonal polynomials”. Useful trick: generating functions.

2.2. Systems of eigenfunctions of Hermitian operators in various linear Euclidean spaces of functions. Orthogonality and completeness of various trigonometric series. Study of examples of expansion.

2.3. Discussion and study: which special functions are included into various software packages.

3. Fourier series (in narrow sense), Fourier integrals.

The purpose of this section is to refresh the knowledge of Fourier Transformations. In particular, the following thesis will be emphasized: “Fourier lives on singularities”.
3.1. Fourier transformations from function $g(x)$ to $h(k)$, from $g(t)$ to $h(\omega)$, which uses $\exp(ikx)$ or $\exp(-i\omega t)$, with inverted dimensions: $[k] = [1/x]$, $[\omega] = [1/t]$. Variants to distribute the $(1/2\pi)$ coefficient.

3.2. Fourier operator F acting as

$$h(y) \equiv \mathsf{F}[g](y) = (1/2\pi)^{1/2} \int \exp(iyy')\, g(y')\mathrm{d}\, y',$$

with functions $g(y)$, $h(y)$ being the functions of the same dimensionless argument y and belonging to the same (infinite-dimensional) linear space. Properties

$$\mathsf{F}^{-1} = \mathsf{F}^{+}, \qquad \mathsf{F}^{4} = 1$$

of that operator. Numerous eigenfunctions of that operator, including Hermit-Gaussian modes / quantum oscillator wave functions. Hyperbolic secant as an eigenfunction of that operator.

3.3. Integration by parts as the way to elucidate the contributions of singularities. Notions of edge waves. Apodization. Singularities in complex plane will be studied later, in Section 6.

3.4. Notion of discrete Fourier transform the particular finite-dimensional unitary transformation.

3.5. Use of software packages to calculate Fourier Transform. Advantages and dangers of Fast Fourier Transform (FFT) implementations, including those in various software packages. Advantage: fast. Danger: re-mapping of the interval, easy for mathematicians, may be clumsy for consumers (for us).

4. Ordinary Differential Equations (ODEs).

The purpose of this section is to refresh the knowledge of the solving methods for the most frequently encountered types of ODEs.

4.1. Linear ODEs without right-hand-side.

4.1.1. Number of linearly independent solutions; analogs of Wronsky's theorem. Physical analogies: conservation of brightness; preservation of phase space, Lagrange-Helmholtz invariant, Second Law of Thermodynamics.

4.1.2. Equations with constant coefficients; see Lagrange Interpolation Formula from Section 1.4.

4.1.3. 1-d wave equation and WKB approach (approximation) to solving it.

4.1.4. Equations describing parametric resonance and their approximate solutions. Step aside into electrodynamics: role of impedance.

4.2 Equation

$$\mathrm{d}a/\mathrm{d}t + [\Gamma(t) + i\omega(t)]a(t) = f(t).$$

and its most general solution.

4.3. General (nonlinear) ODEs.

4.3.1 The case when all boundary conditions are prescribed at one end, i.e. Cauchy problem. Use of software to solve Cauchy problem for a system of ODEs.

4.3.2. Boundary conditions are distributed between different points: what to do ? Separate linear and nonlinear ODEs.

4.3.3. "Analytically solvable" nonlinear ODEs: DEs with separable variables.

4.4. Use of software packages to solve Cauchy problem.

Note to the material of Section 4. Invariance of DEs with respect to some or other transformations of coordinates and functions helps to solve these DEs and often leads to meaningful conservation laws, like energy, pressure flux, etc.

5. Partial Differential Equations (PDEs).

The purpose of this section is to discuss the frequently encountered PDEs and their solutions. General view of "characteristics" of PDEs will be discussed as well.

5.1. Preliminary info: integration by parts in *n*-dimensional integrals.

5.2. Helmholtz Equation (HE):

$$[\,(\nabla\cdot\nabla) + k^2(\mathbf{r})\,]\,\mathrm{u}(\mathbf{r}) = 0.$$

5.2.1. Spatially homogeneous case.

5.2.2. General solution with the use of Green's function: exact and approximate Huygens' principle.

5.2.3. Parabolic / paraxial approximation; estimation of the accuracy of that approximation.

5.3. Thermal conductivity equation

5.4. Notion of PDE's "characteristics" and their relationship to Fourier expansion.

5.5. Lorentz-Invariance (LI) of D'Alembert equation is almost evident – hence LI of Maxwell equations.

6. Analytic Functions of Complex Variable.

The purpose of this section is not (repeat, not) to show the beauty of underlying mathematics. To the contrary, the purpose is to help solving particular ODEs and PDEs and to provide technical tools for asymptotically calculating certain integrals.

6.1. Definition; independence of integral upon the contour; poles and residues.

6.2. Stirling's formula for *n*!

6.3. Special functions (Airy, Bessel, and other) as solutions of ODEs. “Stitching” and choice out of two linearly independent solutions in different regions of argument.

6.4. Steepest descent method of asymptotic calculation of integrals.
 6.4.1. Critical point is at the boundary.
 6.4.2. Critical point is inside the integration interval.

6.5. Asymptotic solution of the problem of adiabatic passage (Landau-Zener theory).

Time permitting:

6.6. Dispersion relations

6.7. Tunnel effect and Frantz-Keldysh effect: asymptotic theory. Keldysh theory of multiphoton / tunnel ionization.

Necessary topic (probably at the beginning of the course): Newton’s binomial formula

$$(1 + x)^\alpha = 1 + \alpha x + \alpha(\alpha-1)x^2/2! + \alpha(\alpha-1)(\alpha-2)x^3/3! + \alpha(\alpha-1)(\alpha-2)(\alpha-3)x^4/4! + \ldots$$

for arbitrary (positive/negative, integer/fractional, real/complex) values of parameter α. Power series for other functions (to be known by heart): $\exp(x)$, $\ln(1-x)$, $\sin(x)$, $\cos(x)$, $\sinh(x)$, $\cosh(x)$, $\arctan(x)$, $\text{arctanh}(x)$. Formulas for the sum of arithmetic progression, for the sum of geometric progression.