Lecture 23

Stimulated Raman scattering, Stokes and anti-Stokes waves. Stimulated Brillouin scattering.

Stimulated Raman scattering (SRS)
Stimulated Raman scattering (SRS)

We will see now how energy dissipation can lead to both field attenuation and field amplification.

\[ \text{Molecule: vibrates at freq. } \Omega \]

\[ \omega \pm \Omega \text{ sidebands} \]

\[ \omega \]
Stimulated Raman scattering (SRS)

Two laser frequencies force molecule to vibrate at freq. $\Omega$

Dipole moment of a molecule: $\mathbf{p} = \varepsilon_0 a \mathbf{E}$

key assumption of the theory:

$\alpha = a_0 + \frac{\partial \alpha}{\partial q} q$

Energy due the oscillating field $\mathbf{E}$:

$W = \frac{1}{2} \varepsilon_0 \alpha B^2$

The applied optical field exerts a force

$F = \frac{dW}{dq} \equiv \frac{1}{2} \varepsilon_0 \left( \frac{\partial \alpha}{\partial q} \right) E^2$

This modulated intensity coherently excites the molecular oscillation at frequency $\omega_L - \omega_S = \Omega$. 

Total field squared time averaged:

$\langle \mathbf{E}_1 \cos(\omega t) + \mathbf{E}_2 \cos(\omega - \Omega)t \rangle^2$

$\dot{I}(t) = I_0 + I_1 \cos(\omega_L - \omega_S)t$. 

$\mathbf{E}(\text{freq. } \Omega)$

$q-\text{ normal coordinte}$
Stimulated Raman scattering (SRS)

Total field
\[ E(t) = \frac{1}{2} (E_L e^{i(\omega_L - k_L x)} + E_S e^{i(\omega_S - k_S x)} + c.c. ) \]

Force term oscillating at \( \Omega \) is:
\[ F(z, t) = \frac{1}{2} \varepsilon_0 \left( \frac{\partial^2 \alpha}{\partial q^2} \right) \frac{1}{4} E_L E_S^* e^{i(\Omega t - k x)} + c.c. \]

Simple oscillator model for molecular motion.
\[ \ddot{q} + \gamma \dot{q} + \Omega_0^2 q = F(t) \]

Look for \( q \) in the form
\[ q = \frac{1}{2} [q(\Omega) e^{i(\Omega t - k x)} + c.c.] \]

We thus find that the amplitude of the molecular vibration is given by
\[ q(\Omega) = \frac{1}{2m} \varepsilon_0 \left( \frac{\partial^2 \alpha}{\partial q^2} \right) \frac{E_L E_S^*}{\Omega_0^2 - \Omega^2 + i\omega} \]

Stimulated Raman scattering (SRS)

NL polarization
\[ P_s(z, t) = \frac{1}{2} \varepsilon_0 N \left( \frac{\partial^2 \alpha}{\partial q^2} \right) q(\Omega) \frac{E_L E_S^*}{\Omega_0^2 - \Omega^2 + i\omega} \]

The part of this expression that oscillates at frequency \( \omega_S \) – the Stokes polarization is given by
\[ P_s(z, t) = \frac{1}{2} \varepsilon_0 N \left( \frac{\partial^2 \alpha}{\partial q^2} \right) q(\Omega) \frac{E_L E_S^*}{\Omega_0^2 - \Omega^2 + i\omega} \]

Fourier component of NL polarization
\[ P(\omega) = \frac{1}{2} \varepsilon_0 N \left( \frac{\partial^2 \alpha}{\partial q^2} \right) q(\Omega) \frac{E_L E_S^*}{\Omega_0^2 - \Omega^2 + i\omega} \]
Stimulated Raman scattering (SRS)

Now plug Fourier component into SVEA equation:

$$\frac{\partial E(\omega_j)}{\partial z} = -\frac{i\omega_j}{2n\varepsilon_0} P_{NL}(\omega_j)$$

to get:

$$\frac{dE(\omega_S)}{dz} = -\frac{i\omega_S}{8n\varepsilon_0} \frac{\epsilon_0^2 N}{m} \left( \frac{\partial a}{\partial q} \right)^2 |E_L|^2 E(\omega_S) e^{-\Delta k z}$$

Note that $\Delta k = 0$ here -- SRS is always phase matched, since

$$\Delta k = k_L - k_S + k_S = 0$$

Thus Stokes wave can propagate in any direction, even counter-propagate!

Let us now set $\Omega = \Omega_0$

$$\frac{dE(\omega_S)}{dz} = -\frac{i\omega_S}{8n\varepsilon_0} \frac{\epsilon_0^2 N}{m} \left( \frac{\partial a}{\partial q} \right)^2 |E_L|^2 E(\omega_S) = \frac{1}{8n\varepsilon_0} \frac{\epsilon_0^2 N}{m} \gamma |E_L|^2 |E(\omega_S)|$$

At exact resonance, purely imaginary

$$\gamma = \gamma_{21}$$

$$\frac{dE(\omega_S)}{dz} = aE(\omega_S) \quad \text{exponential growth}$$

---

Stimulated Raman scattering (SRS)

How wide is Raman resonance?

$\gamma_{21}$- inverse phonon lifetime

Lorentz function

$|\text{Im}[\chi^{(3)}]|$

$2\gamma_{12}$

$\omega_L - \omega_S$ detuning from resonance

---

SRS gain bandwidth: from GHz to THz

Raman freq. shift: 15.6 THz in silicon
120 THz in H₂
Stimulated Raman scattering (SRS)

Quantum description

Nonlinear susceptibility $\chi^{(3)}$, quantum mechanical model

According to Boyd, Stegeman

$\chi^{(3)}$ is a huge sum of many different terms

$$
\chi_{ijjk}(\omega_p, \omega_q, \omega_r, \omega_p) = \frac{N}{\epsilon_0 \hbar^3} \mathcal{P}_F \sum_{mnv} \frac{\mu_{ij}^v \mu_{ln}^m \mu_{mn}^v}{(\omega_v - \omega_l)(\omega_m - \omega_n)(\omega_{ln} - \omega_p)}, \quad (3.2.33)
$$

where $\omega_v = \omega_p + \omega_q + \omega_r$ and where we have made use of the full permutation operator $\mathcal{P}_F$ defined following Eq. (3.2.28).

- here frequencies $\omega_p$, $\omega_q$, $\omega_r$ can be both positive and negative and can each take values of $\pm \omega_1, \pm \omega_2, \pm \omega_3, \pm \omega_4$
Stimulated Raman scattering (SRS)

Let us assume we have only two frequencies ('Laser' and 'Stokes') \( \omega_L \) and \( \omega_S \) \( (\omega_S < \omega_L) \) and take the terms that are close to the Raman resonance, that is \( \omega_{21} \) is close to \( \omega_L - \omega_S \)

\[
\chi^{(3)} = \frac{N}{\varepsilon_0^2} \mu_{14} \mu_{12} \mu_{23} \mu_{34} \\
\times \sum_{p,q,r} \frac{1}{[(\omega_{21}-(\omega_L - \omega_S))(\omega_{41}-\omega_p - \omega_q - \omega_r)(\omega_{31}-\omega_p)^+\cdots]}
\]

resonant term - close to zero

\[
\rightarrow \sum_{p,q,r} \frac{N}{\varepsilon_0^2} \mu_{14} \mu_{12} \mu_{23} \mu_{34} \\
\times \frac{1}{[(\omega_{21}-(\omega_L - \omega_S) + i\gamma_{21}[(\omega_{41}-\omega_p - \omega_q - \omega_r)(\omega_{31}-\omega_p)^+\cdots]]}
\]

damping term added

Stimulated Raman scattering (SRS)

We have two waves \( \omega_L \quad \omega_S \quad (\omega_L > \omega_S) \quad E_L \gg E_S \)

Regard a 4-wave process \( \omega_S = \omega_L - \omega_L + \Delta \omega \)

total field \( E(t) = \frac{1}{2} (E_L e^{i\omega_L t} + E_S e^{i\omega_S t} + c.c.) \)

total NL polariz. \( P_{NL}(t) = \frac{1}{8} \varepsilon_0 X^{(3)}(E_L e^{i\omega_L t} + E_S e^{i\omega_S t} + c.c.)^3 \)

Now let us pick only components with \( \pm \omega_S \) - due to interaction of two waves

\[
P_{NL}(\omega_S) = \frac{3}{2} \varepsilon_0 X^{(3)} E_L E_S e^{i\omega_S t}
\]

Fourier component

\( P(\omega_S) = \frac{3}{2} \varepsilon_0 X^{(3)} E_L E_S \)
Stimulated Raman scattering (SRS)

Now plug into SVEA equation:

\[
\frac{dE(\omega_j)}{dz} = -\frac{i \omega_j}{2nc^2} P_{NL}(\omega_i)
\]

\[
\frac{dE(\omega_S)}{dz} = \frac{3 \omega_0 \chi^{(3)}(j)}{2n} |E_L|^2 E(\omega_S) = \frac{3 \omega_0 \chi^{(3)}(j)}{2n} \frac{2L_r}{\epsilon_0} E(\omega_S) = \frac{3 \omega_0 \chi^{(3)}(j)}{2n} \frac{2L_r}{\epsilon_0} \frac{n^2 c^2}{\omega} I_S
\]

If we set:

\[\chi^{(3)} = \chi^{(3)}_R + i \chi^{(3)}_I\]

- real and imaginary parts

Real part - responsible for cross-phase modulation
Imaginary part - responsible for Raman gain

For intensity:

\[
\frac{dI_S}{dz} = g_R I_L I_S
\]

\[I_S = I_{0,0} e^{g_R L}\]

\[g_R = \frac{3 \omega_0 \chi^{(3)}(j)}{2n^2 \epsilon_0 c^2} \text{ measured in cm/W}
\]

Raman gain coeff.

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Stimulated Raman scattering (SRS)

**Table 10.2.1 Properties of stimulated Raman scattering for several materials**

<table>
<thead>
<tr>
<th>Substance</th>
<th>Frequency Shift (\nu_0) (cm(^{-1}))</th>
<th>Linewidth (\Delta \nu) (cm(^{-1}))</th>
<th>Cross Section (\sigma(\omega, \omega_0)) (10(^{-26}) m(^2) s(^{-1}))</th>
<th>Gain Factor (g_c / L) (m(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid O(_2)</td>
<td>1552</td>
<td>0.117</td>
<td>0.48 ± 0.14</td>
<td>145</td>
</tr>
<tr>
<td>Liquid N(_2)</td>
<td>2326.5</td>
<td>0.067</td>
<td>0.29 ± 0.09</td>
<td>160 ± 50</td>
</tr>
<tr>
<td>Benzene</td>
<td>992</td>
<td>2.15</td>
<td>3.06</td>
<td>28</td>
</tr>
<tr>
<td>C(_6)H(_6)</td>
<td>655.6</td>
<td>0.50</td>
<td>7.55</td>
<td>240</td>
</tr>
<tr>
<td>N(_2)O(_2)</td>
<td>1345</td>
<td>6.6</td>
<td>6.4</td>
<td>21</td>
</tr>
<tr>
<td>Dry Nitrogen</td>
<td>1000</td>
<td>1.9</td>
<td>1.5</td>
<td>15</td>
</tr>
<tr>
<td>Chloroform</td>
<td>1002</td>
<td>1.6</td>
<td>1.5</td>
<td>19</td>
</tr>
<tr>
<td>Toluene</td>
<td>1003</td>
<td>1.94</td>
<td>1.1</td>
<td>12</td>
</tr>
<tr>
<td>N(_2)O(_3)</td>
<td>256</td>
<td>23</td>
<td>381</td>
<td>89</td>
</tr>
<tr>
<td>Dry Nitrogen</td>
<td>637</td>
<td>20</td>
<td>231</td>
<td>94</td>
</tr>
<tr>
<td>Dry Nitrogen</td>
<td>650</td>
<td>22</td>
<td>238</td>
<td>44</td>
</tr>
<tr>
<td>Li(_2)O(_2)</td>
<td>201</td>
<td>22</td>
<td>238</td>
<td>44</td>
</tr>
<tr>
<td>SiO(_2)</td>
<td>467</td>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>Methane gas</td>
<td>2916</td>
<td></td>
<td>(10 atm)</td>
<td>6.6</td>
</tr>
<tr>
<td>H(_2) gas</td>
<td>4155</td>
<td>(&gt; 10 atm)</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>H(_2) gas (rot.)</td>
<td>450</td>
<td>(&gt; 0.5 atm)</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Deuterium gas</td>
<td>2991</td>
<td>(&gt; 10 atm)</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>N(_2) gas</td>
<td>2326</td>
<td>(10 atm)</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>O(_2) gas</td>
<td>1555</td>
<td>(10 atm)</td>
<td>0.16</td>
<td></td>
</tr>
</tbody>
</table>

\(a\) After Kaiser and Maier (1972) and Simon and Titel (1994). All transitions are vibrational except for the 430 cm\(^{-1}\) hydrogen transition which is rotational.

\(b\) Measured at 694 nm unless stated otherwise.

\(c\) Measured at 500 nm.
Stimulated Raman scattering (SRS)

| Properties of Raman media | Silicon | Ba(NO$_3$)$_2$ | LiIO$_3$ | KGd(WO$_4$)$_2$ | CaWO$_4$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission Range (µm)</td>
<td>1.1-6.5</td>
<td>0.38-1.8</td>
<td>0.38-5.5</td>
<td>0.35-5.5</td>
<td>0.2-5.3</td>
</tr>
<tr>
<td>Refractive index</td>
<td>3.42</td>
<td>1.556</td>
<td>1.84 (o)</td>
<td>1.711 (c)</td>
<td>1.884 (o)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.033</td>
<td>1.903</td>
<td>1.898 (c)</td>
</tr>
<tr>
<td>Raman shift at 300K (cm$^{-1}$)</td>
<td>521</td>
<td>1047.3</td>
<td>770</td>
<td>822</td>
<td>903</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>768</td>
<td></td>
<td>910.7</td>
</tr>
<tr>
<td>Spontaneous Raman linewidth (cm$^{-1}$)</td>
<td>3.5</td>
<td>0.4</td>
<td>5.0</td>
<td>5.9</td>
<td>4.8</td>
</tr>
<tr>
<td>Raman gain (cm/GW)</td>
<td>20 (1550nm)</td>
<td>11 (1064nm)</td>
<td>4.8 (1064nm)</td>
<td>3.3 (1064)</td>
<td>-</td>
</tr>
<tr>
<td>Optical damage threshold (MW/cm$^2$)</td>
<td>~1000-1000</td>
<td>~400</td>
<td>~100</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Thermal conductivity (W/m-K)</td>
<td>148</td>
<td>1.17</td>
<td>-</td>
<td>2.6 [1 0 0]</td>
<td>16</td>
</tr>
</tbody>
</table>

Silicon: The gain coefficient extracted from measurements performed near 1550 nm is 20 cm/GW

Stimulated Raman scattering (SRS)


Raman gain in glass optical waveguides

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(Received 20 November 1972)

The small signal Raman gain in a single-mode glass waveguide amplifier has been measured directly. The measured gain is in good agreement with that calculated from the Raman cross section. The cross section was determined by a comparison of the spontaneous Raman scattering of fused quartz and benzene.
Stimulated Raman scattering (SRS)

Fiber: fused quartz (3.8 µm core diam.)
Pump: Xenon laser at λ=526 nm
Signal: at λ=535.3 nm (Raman frequency shift 330 cm⁻¹ or 10 THz)
Measured Raman gain coeff. \( g = 1.5 \times 10^{-11}\) cm/W

Example: for the pump power of 100W \( (I_p = 0.75\) GW/cm²) and \( L=590\) cm we get:

\[
\text{Gain} = \exp(gI_pL) = \exp(6.6) = 740
\]

Stimulated Raman scattering (SRS)

The Raman-active medium is often an optical fiber, although it can also be a bulk crystal, a waveguide in a photonic integrated circuit, or a cell with a gas or liquid medium.

For application in telecom systems, fiber Raman amplifiers compete with erbium-doped fiber amplifiers. Compared with those, their typical features are:

- Raman amplifiers can be operated in very different wavelength regions, provided that a suitable pump source is available.
- The gain spectrum can be tailored by using different pump wavelengths simultaneously.
- A Raman amplifier requires high pump power (possibly raising laser safety issues) and high pump brightness, but it can also generate high output powers.
- A greater length of fiber is required. However, the transmission fiber in a telecom system may be used, so that no additional fiber is required.
- Raman fiber amplifiers can have a lower noise figure. On the other hand, they more directly couple pump noise to the signal than laser amplifiers do.
- They also have a fast reaction to changes of the pump power, particularly for co-propagating pump, and very different saturation characteristics.
- If the pump wavelength is polarized, the Raman gain is polarization-dependent. This effect is often unwanted, but can be suppressed e.g. by using two polarization-coupled pump diodes or a pump depolarizer.

A telecom Raman amplifier is pumped with continuous-wave light from a diode laser.
Stimulated Raman scattering (SRS)

Extending DWDM Network Reach With Raman Amplifier

Raman amplifier is appearing to be a critical technology which is consistently developed for using in optical communication networks. Typically applied in long-haul networks, Raman amplifier is also expected to extend its reach in dense wavelength-division multiplexing (DWDM) networks. This escalating adoption, therefore, is fueled by the massive bandwidth demand that network operators are continuously facing. This article explains the necessities and related considerations for deploying Raman amplifier in DWDM networks.

Why Use Raman Amplifier and How it Works?

Raman amplifier has proved itself beneficial for applications in 100G network and above. It is gaining in popularity because it is capable of meeting the need for higher transmission capacity. There exist various alternatives to enhance network transmission capacity: like extending beyond the C-band into the L-band, increasing the symbol rate or increasing spectral efficiency. Any of the options requires a higher optical signal-to-noise ratio (OSNR). Raman amplifier generally offers higher OSNR required to increase capacity, while eliminates the need for expensive opto-electronic regeneration.

A continuous-wave Raman silicon laser

A continuous-wave Raman silicon laser

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Achieving optical gain and/or lasing in silicon has been one of the most challenging goals in silicon-based photonics because bulk silicon is an indirect bandgap semiconductor and therefore has a very low light emission efficiency. Recently, stimulated Raman scattering has been used to demonstrate light amplification and lasing in silicon. However, because of the nonlinear optical loss associated with two-photon absorption (TPA)-induced free carrier absorption (FCA), until now lasing has been limited to pulsed operation. Here we demonstrate a continuous-wave silicon Raman laser. Specifically, we show that TPA-induced FCA in silicon can be significantly reduced by introducing a reverse-biased p-i-n diode embedded in a silicon waveguide. The laser cavity is formed by coating the facets of the silicon waveguide with multilayer dielectric films. We have demonstrated stable single mode laser output with side-mode suppression of over 55 dB and linewidth of less than 80 MHz. The lasing threshold depends on the p-i-n reverse bias voltage and the laser wavelength can be tuned by adjusting the wavelength of the pump laser. The demonstration of a continuous-wave silicon laser represents a significant milestone for silicon-based optoelectronic devices.
A continuous-wave Raman silicon laser

Pump wavelength 1550 nm.
Raman/Stokes wavelength 1686 nm
The effective core area (1.6 µm)^2
WG length: 4.8 cm
When a reverse bias voltage (~25V) is applied, the TPA-generated electron–hole pairs can be swept out of the silicon waveguide by the electric field between the p- and n-doped regions.
Raman gain was measured in a pump–probe experiment: a single-pass gain of 3 dB (~2^3) at a pump power of 700mW coupled into the waveguide.

Figure 1: Silicon waveguide used in the Raman laser experiment. a. Schematic layout of the silicon waveguide laser cavity with optical coatings applied to the facets and a p-i-n structure along the waveguide. b. Scanning electron microscope cross-section image of a silicon rib waveguide with a p-i-n diode structure.

A continuous-wave Raman silicon laser

The Raman laser frequency shift is 15.6 THz (520 cm⁻¹)
The lasing threshold was 180mW with a 25-V bias.

Figure 3: Silicon Raman laser output power as a function of the input pump power at a reverse bias of 25 and 5 V. The pump wavelength is 1,550 nm and the laser wavelength is 1,666 nm. The slope efficiency (single side output) is 4.3% for 25-V bias and 2% for 5-V bias. Error bars represent standard deviations.

Monolithic integration of silicon-based optoelectronics
Stimulated Raman scattering (SRS)

Cascaded Raman laser (converter) based on a tellurite fiber and Tm pump laser.

Coherent Anti-Stokes Raman Scattering
Coherent Anti-Stokes Raman Scattering (CARS)

- Can be regarded as a resonant 4-wave mixing process

\[ \omega_{AS} = \omega_{L} - \omega_{S} + 2\omega_{L} = \omega_{S} \]

Phase matching is critical

\[ k_{AS} = 2k_{L} - k_{S} \]

Collinear phase matching is impossible, but vector phase matching is possible; hence the outputs are in the form of a cone.

**Figure 10.3.5**: Phase-matching relations for Stokes and anti-Stokes coupling in stimulated Raman scattering.

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CARS microscopy
Stimulated Brillouin scattering (SBS)

Brillouin scattering is a light–sound interaction process that occurs when photons are scattered from a medium by induced acoustic waves. Stimulated Brillouin Scattering (SBS) manifests itself in narrow resonances.

In contrast to optical phonons that are stationary, acoustic waves travel at the velocity of sound.

The damping of acoustic phonons at the frequencies typical of SBS (tens of gigahertz) is large with decay lengths less than 100 µm.
Stimulated Brillouin scattering (SBS)

Only backward traveling Stokes and anti-Stokes waves can occur. The speed of sound is $\sim 10^5$ times less than that of light.

Stimulated Brillouin scattering (SBS)

The evolution of the pump beam in the forward direction and the Stokes beam in the backward direction.
Stimulated Brillouin scattering (SBS)

Exponential Growth: \[ I_\infty = I_{SOE} g_B L \]

\[ g_B^{\max} = \frac{\omega_0 \hbar^2 \rho_{12}^2 \Omega}{4 \rho v_s^2 \tau_s c^2} \]

- \( \tau_s \) is the decay time for the sound wave
- \( \rho_{12} \) is the acousto-optic constant \( 0.1 < \rho_{12} < 1 \)
- \( \Omega \) - frequency of sound
- \( \rho \) - density
- \( v_s \) - speed of sound

SBS is the dominant nonlinear effect for continuous-wave beams.

Example: fused silica.
\( \lambda = 1.55 \text{ µm}, n = 1.45, v_s = 6 \text{ km/s}, \Omega/2\pi = 11 \text{ GHz}, 1/\tau_s = 17 \text{ MHz} \)
- get \( g_B = 5 \times 10^{-11} \text{ m/W} = 5 \times 10^{-9} \text{ cm/W} = 5 \text{ cm/GW} \)

This value is 500 times larger than the \( g_R \) value. However, \( 1/\tau_s \) is quite small (the line is narrow) and SBS requires single frequency input.