Lecture 10

Conversion efficiency of the nonlinear optical processes; focused beams, pulsed beams; advantages of the waveguides.

Plane waves vs focused beams

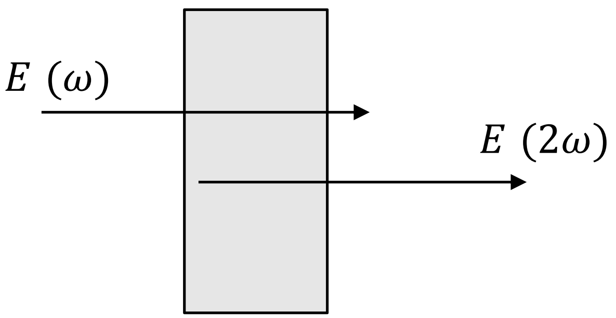
So far we have treated nonlinear optical interactions in the approximation of infinite plane waves (no XY dependence).

However, in practice, the incident radiation is usually focused into the nonlinear optical medium in order to increase its intensity and hence to increase the efficiency of the nonlinear optical process.

This Lecture explores the nature of nonlinear optical interactions that are excited by focused laser beams.

Plane waves vs focused beams: SHG example

In Lecture 6, we derived (for low conversion limit) phase-matched SHG intensity



SH intensity

$$I_{2\omega} = \frac{2\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right) {I_\omega}^2 L^2 \qquad (6.10a)$$

-grows quadratically with distance
-grows quadratically with I_ω

SHG conversion efficiency (plane-wave limit)

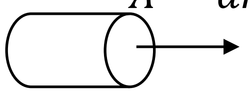
$$\eta_{2\omega} = I_{2\omega}/I_\omega = \frac{2\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right) I_\omega\ L^2 \qquad (6.10b)$$

d – effective NLO coefficient

In practice, we are interested in power conversion efficiency or energy conversion efficiency.

Plane waves vs focused beams: SHG example

Imagine, we have a beam with the average power P_ω
How much power $P_{2\omega}$ at the second harmonic we can get?



Assume we have a top-hat beam at ω with the area A. And **ignore diffraction**, so that the generated beam at 2ω has the same area (and shape) – the so called near-field approximation.

Intensity (power density) at ω $\quad I_\omega = P_\omega / A$

$$P_{2\omega} = I_{2\omega} A = \frac{2\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right) {I_\omega}^2 L^2 A = \frac{2\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\left(\frac{P_\omega}{A}\right)^2 L^2 A = \frac{2\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{P_\omega^2}{A} L^2 \qquad (10.1)$$

$$\eta_{2\omega}^{power} = P_{2\omega}/P_\omega = \frac{2\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{P_\omega}{A} L^2 = \frac{2\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{P_\omega}{A} L^2 \qquad (10.2)$$

Hence focusing (smaller area A) increases conversion efficiency!

Plane waves vs focused beams: SHG example

Rewrite the formula for conversion efficiency and include the phase matching factor :

$$\eta_{2\omega}^{power} = \frac{2\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) I_\omega \, L^2 \; sinc^2\left(\frac{\Delta k L}{2}\right) \qquad (10.2b)$$

Power conversion efficiency

inceases with frequency squared

phase matching factor, unity at Δk = 0.

just fundam. constants

NLO figure of merit (FOM)

power density

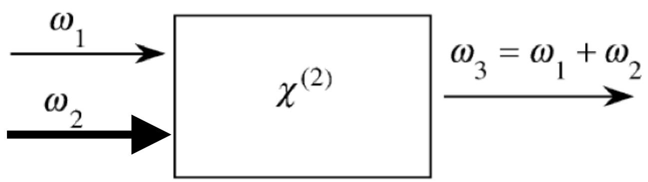
L^2 dependence

This result holds in the plane-wave top-hat beam

Focused beams: SFG example

Repeat the same formalism, but start from SFG equations for a plane wave (Lecture 6).

Asume no absorption, and that the 'pump' field at ω_2 is strong : $\boldsymbol{E_2} \gg \boldsymbol{E_2}$ & $\boldsymbol{E_3}$



$$\eta_{SFG} = \frac{I_{\omega_3}}{I_{\omega_1}} = \frac{2\omega_3^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) I_{\omega_2}\ L^2\ \ sinc^2\left(\frac{\Delta kL}{2}\right) \qquad (10.3)$$

SFG power (intensity) conversion efficiency (low limit), $\omega_1 \rightarrow \omega_3$

ω_3^2 here

just constants

NLO figure of merit (FOM)

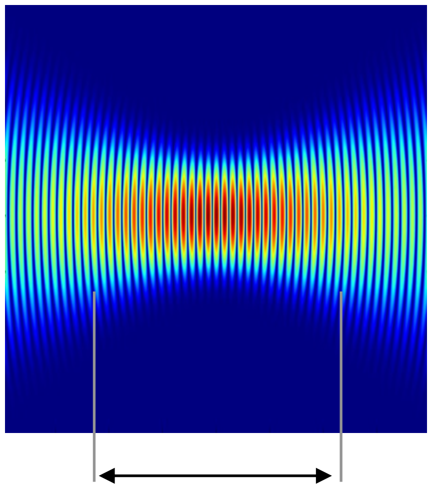
'pump' power density at ω_2

L^2 dependence

This result holds in the plane-wave top-hat beam

Gaussian beams

The beams are usually focussed into the nonlinear crystal to maximize the conversion efficiency. The waves generated by laser sources have a Gaussian amplitude profile with electric field beam radius w.



beam waist length
(focal length b)

$$b = 2Z_R = \frac{2\pi w_0^2}{\lambda}$$

The Gaussian electric field in complex notation is given by:

$$\mathbf{E}(r,z) = E_0\,\hat{x}\,\frac{w_0}{w(z)}\exp\left(\frac{-r^2}{w(z)^2}\right)\exp\left(-i\left(kz + k\frac{r^2}{2R(z)} - \psi(z)\right)\right)$$

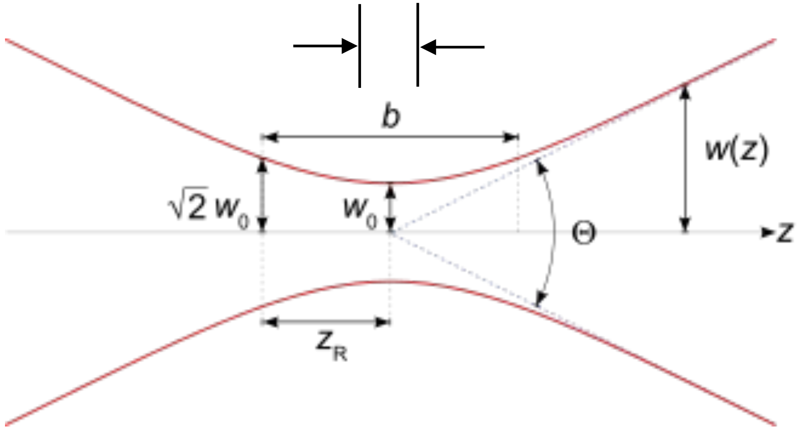
the Gouy phase

R, radius of curvature

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_\mathrm{R}}\right)^2}$$

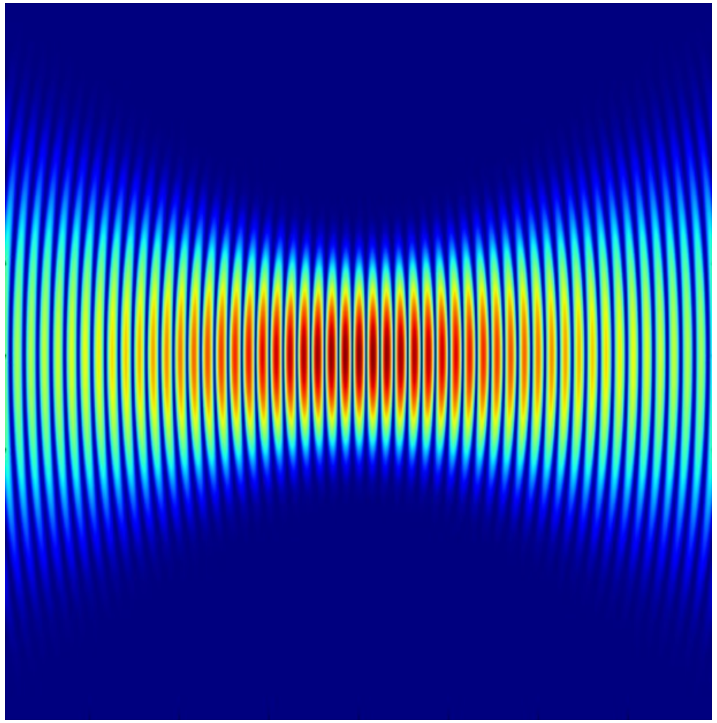
$$z_\mathrm{R} = \frac{\pi w_0^2}{\lambda}$$

near field $z << z_R$



$$E \sim e^{-r^2/w_0^2},\quad I \sim e^{-2r^2/w_0^2}$$

Gaussian beams



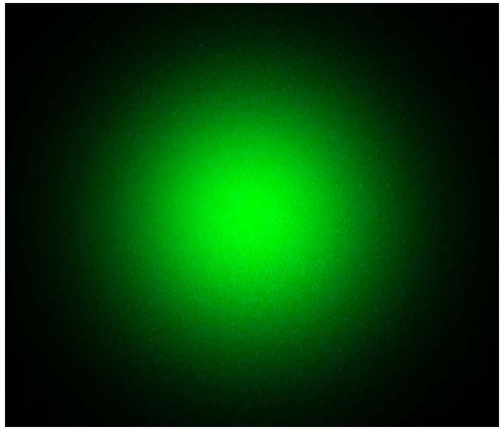
The total power in the Gaussian beam is:

$$P = I_0 \int_0^\infty 2\pi r dr\, e^{-2r^2/w_0^2} = \frac{\pi w_0^2}{2}\, I_0$$

on-axis intensity

efective area of the Gaussian beam A_{eff}

$$I_{max} = P/\, A_{eff}$$

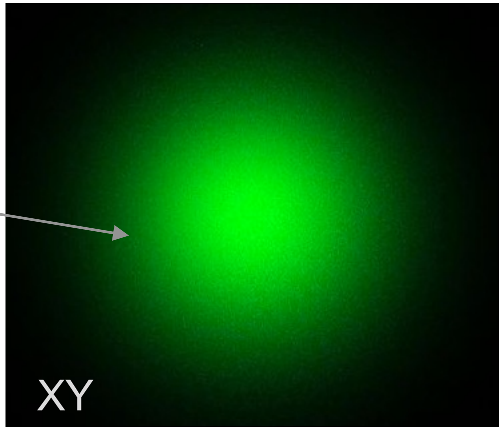


So if we know the power, we know the peak **intensity** (power density)

Gaussian beams, SHG

Let us now calculate SHG **power** conversion efficiency for Gaussian beams in the near field.

Need to integrate over XY plane



Start from:

$$I_{2\omega} = \frac{2\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right) I_\omega{}^2 L^2 \qquad (6.10a)$$

$$P_{2\omega} = \int_0^\infty I_{2\omega}(r) d^2 r = \int_0^\infty I_{2\omega}(r)(2\pi r dr) = \frac{2\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 \int_0^\infty I_\omega(r)^2 (2\pi r dr) =$$

$$= \frac{2\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 \int_0^\infty I_0{}^2\, e^{-4r^2/w_0^2}(2\pi r dr) = \frac{2\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 I_0{}^2 \frac{\pi w_0^2}{4} = \frac{2\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 I_0 \frac{1}{2} \underbrace{\left(\frac{\pi w_0^2}{2} I_0\right)}_{\text{pump power } P_\omega}$$

Gaussian beams

$$P_{2\omega} = P_\omega \frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 I_0 = \frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) \frac{P_\omega^2}{A_{eff}} L^2 \qquad (10.4a)$$

$$\eta_{2\omega}^{power} = P_{2\omega}/P_\omega = \frac{\omega^2}{\epsilon_0 c^3}\left(\frac{d^2}{n^3}\right) \frac{P_\omega}{A_{eff}} L^2 \qquad (10.4b)$$

$$A_{eff} = \frac{\pi w_0^2}{2}$$

This $\frac{1}{2}$ represents averaging in two spatial coordinates for the Gaussian shape

Each coordinate gives $\frac{1}{\sqrt{2}}$ reduction

We will see later that time-domain Gaussian shape gives another $\frac{1}{\sqrt{2}}$ reduction

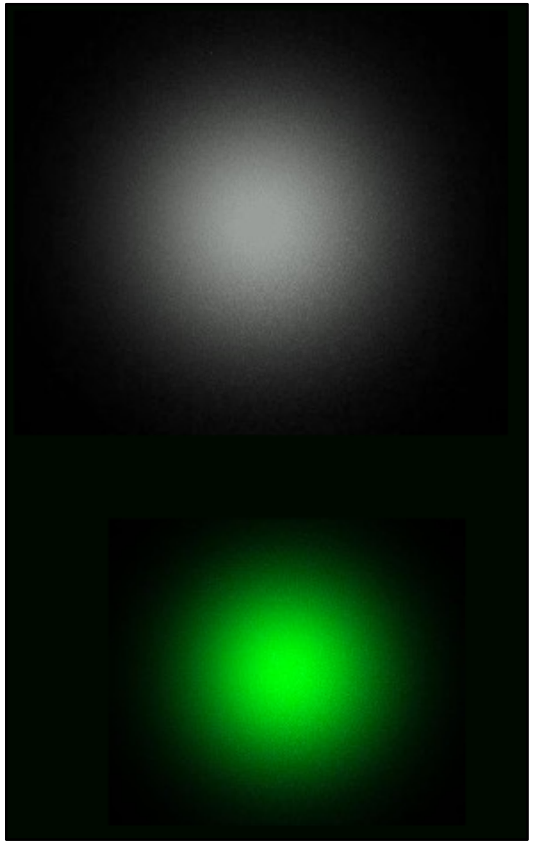
Gaussian beams, SHG

Note that compression of the second harmonic signal occurs in space:

ω $\quad\quad$ 2ω

$$e^{-2r^2/w_0^2} \rightarrow e^{-4r^2/w_0^2}$$

ω



2ω

beamsize reduces by $\sqrt{2}$

Gaussian beams, SHG, numerical examples

$$P_{2\omega} = \frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)L^2\frac{P_\omega^2}{A_{eff}} \qquad (10.4a)$$

Real example

KDP crystal

$\lambda_\omega = 1.06\ \mu m,\ \omega$=1.78e15 s^{-1}
ϵ_0=8.85e-12 F/m
$c = 3e8\ m/s$
$d_{36} = 0.4\ pm/V,$

ooe phase matching, $d_{eff}=d_{36}\sin\theta\sin 2\varphi$
for SHG 1.06->0.53 μm, θ=41º, φ=45º,
d_{eff}= 0.26 pm/V= 0.26e-12 m/V
n=1.5;
L=1 cm (1e-2 m)
w_0=1 mm (Gauss), $A_{eff} = \frac{\pi w_0^2}{2} =$ 1.57e-6 m^2

P_ω=1 W

$P_{2\omega}$= 1.78e15^2/8.85e-12/3e8^3 * (0.26e-12^2/1.5^3) *1e-2^2/1.57e-6 *1^2=
= 17e-9 W = 17 nW

Power conversion efficiency

$\eta_{2\omega}$ =1.7e-8 ~ 10^{-8}

Gaussian beams, SHG, numerical examples

$$P_{2\omega} = \frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{L^2}{A_{eff}}\ P_\omega^2 \qquad (10.4)$$

Real example

PPLN crystal

$\lambda_\omega = 1.06\ \mu m$, ω=1.78e15 s^{-1}
ϵ_0=8.85e-12 F/m
c=3e8 m/s

d_{33}=28 pm/V,
eee quasi phase matching,
$d_{eff}=\frac{2}{\pi}d_{33}$ =18 pm/V=18e-12 m/V
n=2.14;
L=5 cm (5e-2 m)
w_0=90 µm (Gauss), $A_{eff} = \frac{\pi w_0^2}{2}$ =1.27e-8 m^2
P_ω=1 W

$P_{2\omega}$= 1.78e15^2/8.85e-12/3e8^3 *(18e-12^2/2.14^3) *5e-2^2/1.27e-8 *1^2 =

= 0.086 W = 86 mW

Power conversion efficiency

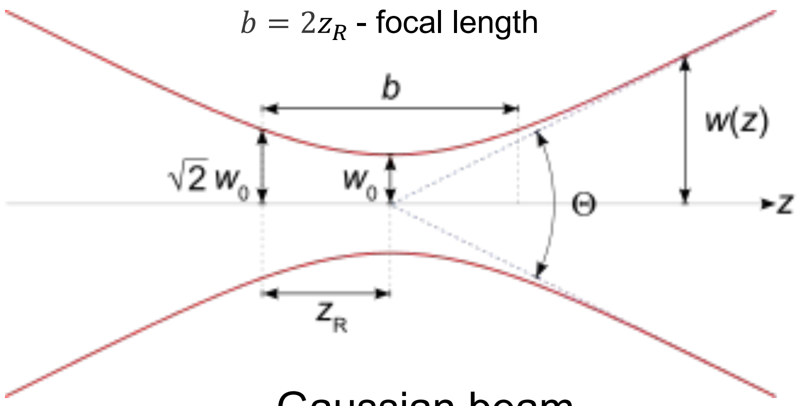
$\eta_{2\omega}$ ~ 0.1

Gaussian beams, SHG

How tightly can we focus the beam ?

From (10.4) it follows that SHG conversion efficiency (at a fixed pump power P_ω) scales as

$$\eta_{2\omega} \sim L^2/w_0^2$$



Gaussian beam

$$I \sim e^{-2r^2/w_0^2}$$

Rayleigh length $z_R = \frac{\pi w_0^2}{(\lambda/n)}$

Focusing improves SHG efficiency as $\sim 1/w_0^2$ till you reach the so called confocal limit when the waist w_0 becomes so smal that the Rayleigh length becomes $z_R < L/2$

The beam intensity does not stay constant over the length of the crystal

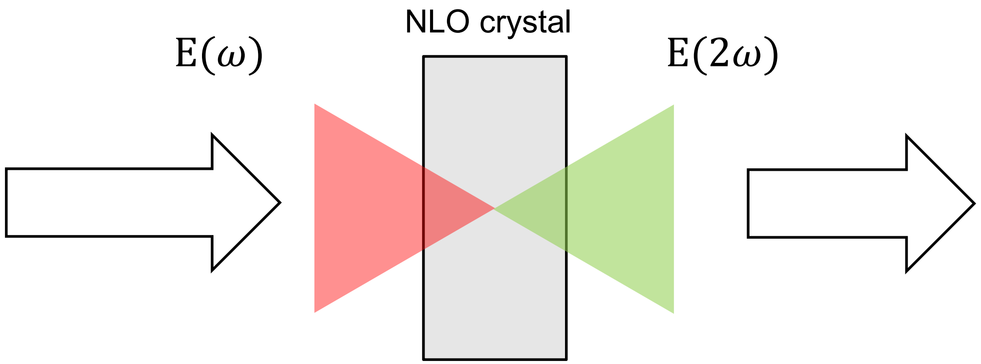
The effective length becomes less than *L*: $L_{eff} \sim 2z_R \sim 2\pi w_0^2/(\lambda/n)$

Hence

$$\eta_{2\omega} \sim L^2/w_0^2 \sim L_{eff}^2/w_0^2 \sim (2z_R)^2/w_0^2 \sim \frac{w_0^4}{w_0^2} \sim w_0^2$$

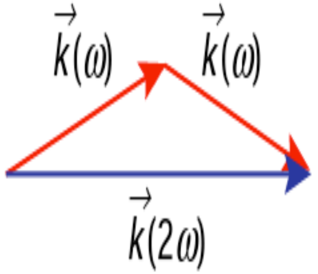
- starts declining at strong focusing $w_0 \to 0$

Gaussian beams, SHG



Very tight focusing is not a good idea:

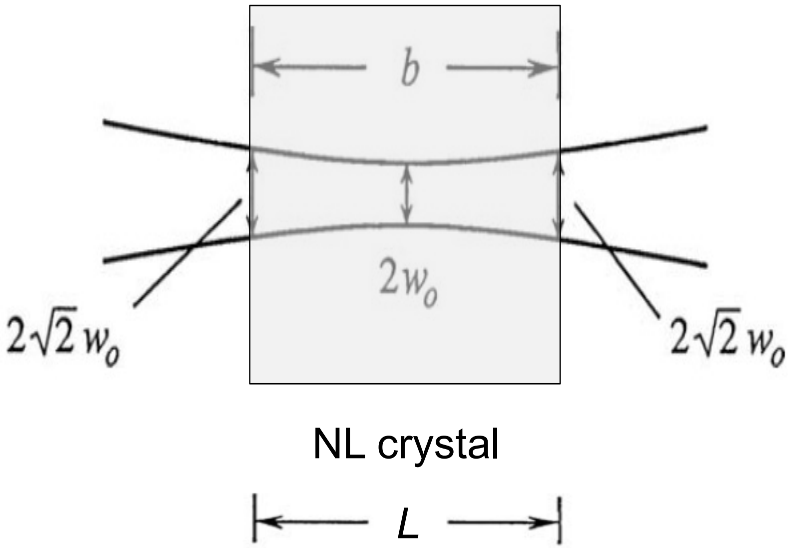
1. Lower efficiency
2. Possibility of crystal damage in the focus
3. Poor phase matching



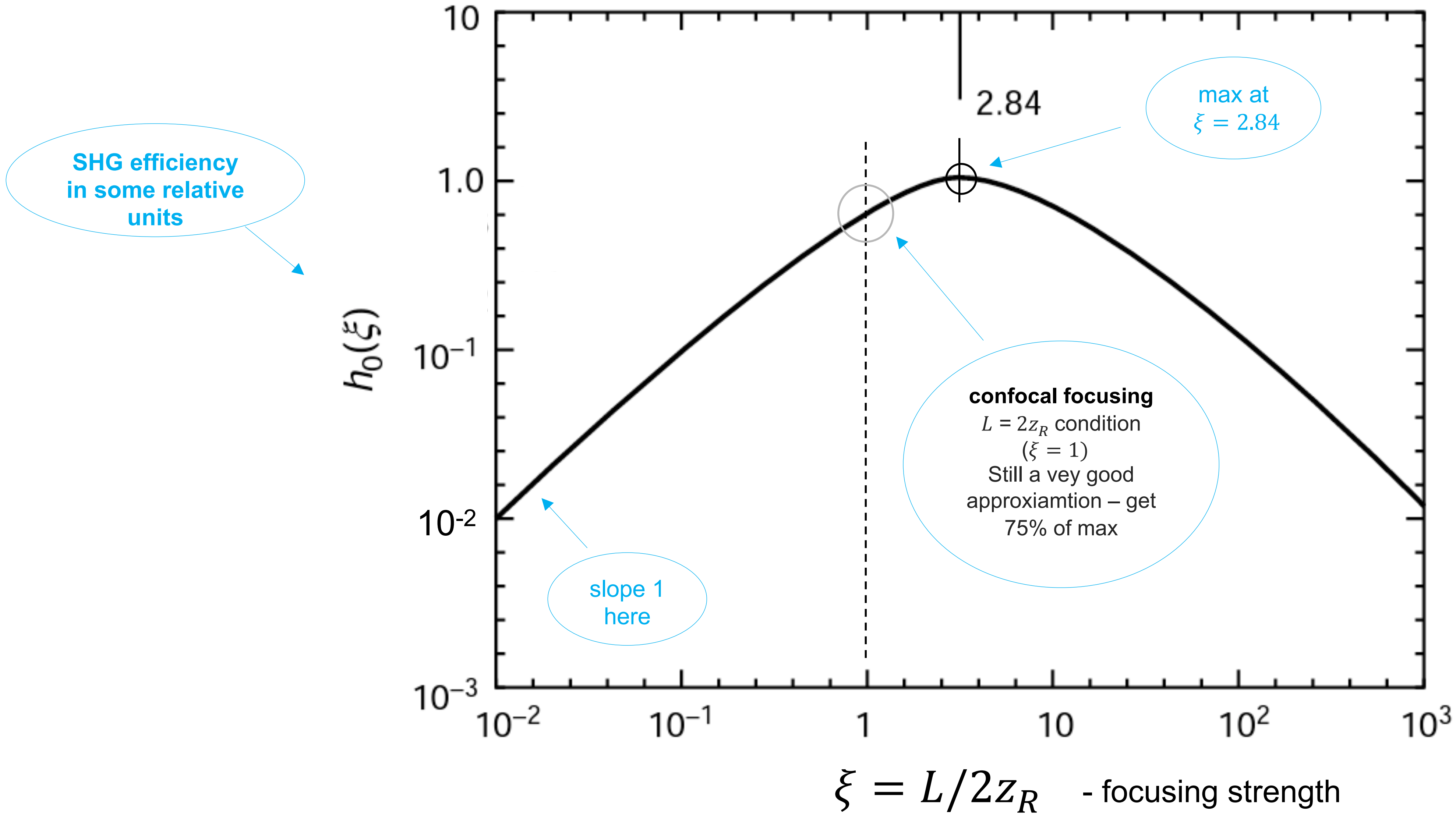
Gaussian beams, SHG

Good approximation: confocal focusing, such that:

$$L = b = 2z_R = 2\pi w_0^2/(\lambda/n)$$



Gaussian beams, SHG optimal focusing after Boyd-Kleinman



At a fixed L, ξ increases with the focusing strength as $1/w_0^2$

G. D. Boyd and D. A. Kleinman, "Parametric interaction of focused Gaussian beams," J. Appl. Phys., **39**, 3597–3639 (1968).

Gaussian beams, SHG

Note that for the **optimized conditions** the second harmonic power is proportional to the sample length *L*, not L^2 .

$$L = 2z_R = \frac{2\pi w_0^2}{\lambda/n} \sim w_0^2$$

optimized focusing

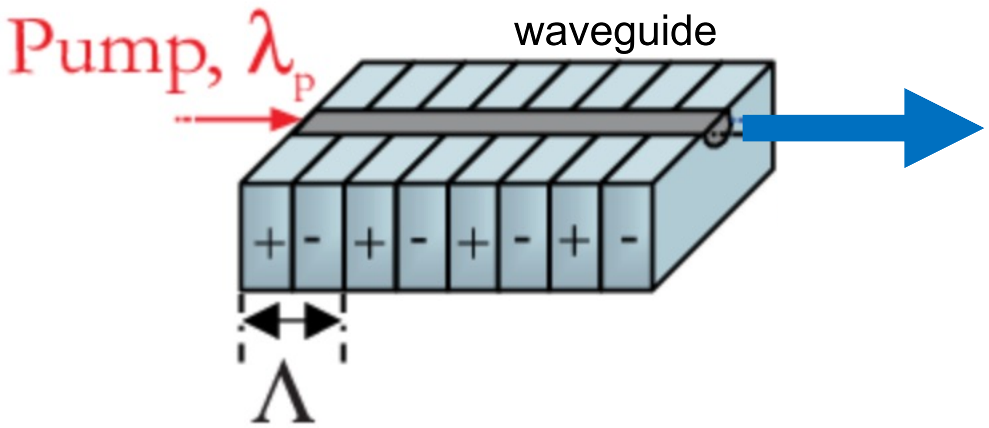
$$\rightarrow \quad w_0^2 \sim L$$

$$\eta_{2\omega} \sim L^2/w_0^2 \sim L^2/L \sim L$$

Gaussian beams, SFG

SFG analysis for Gausian beams is pretty similar to that of SHG, if the frequencies ω_1 and ω_2 are not far from each other

NLO processes in waveguides

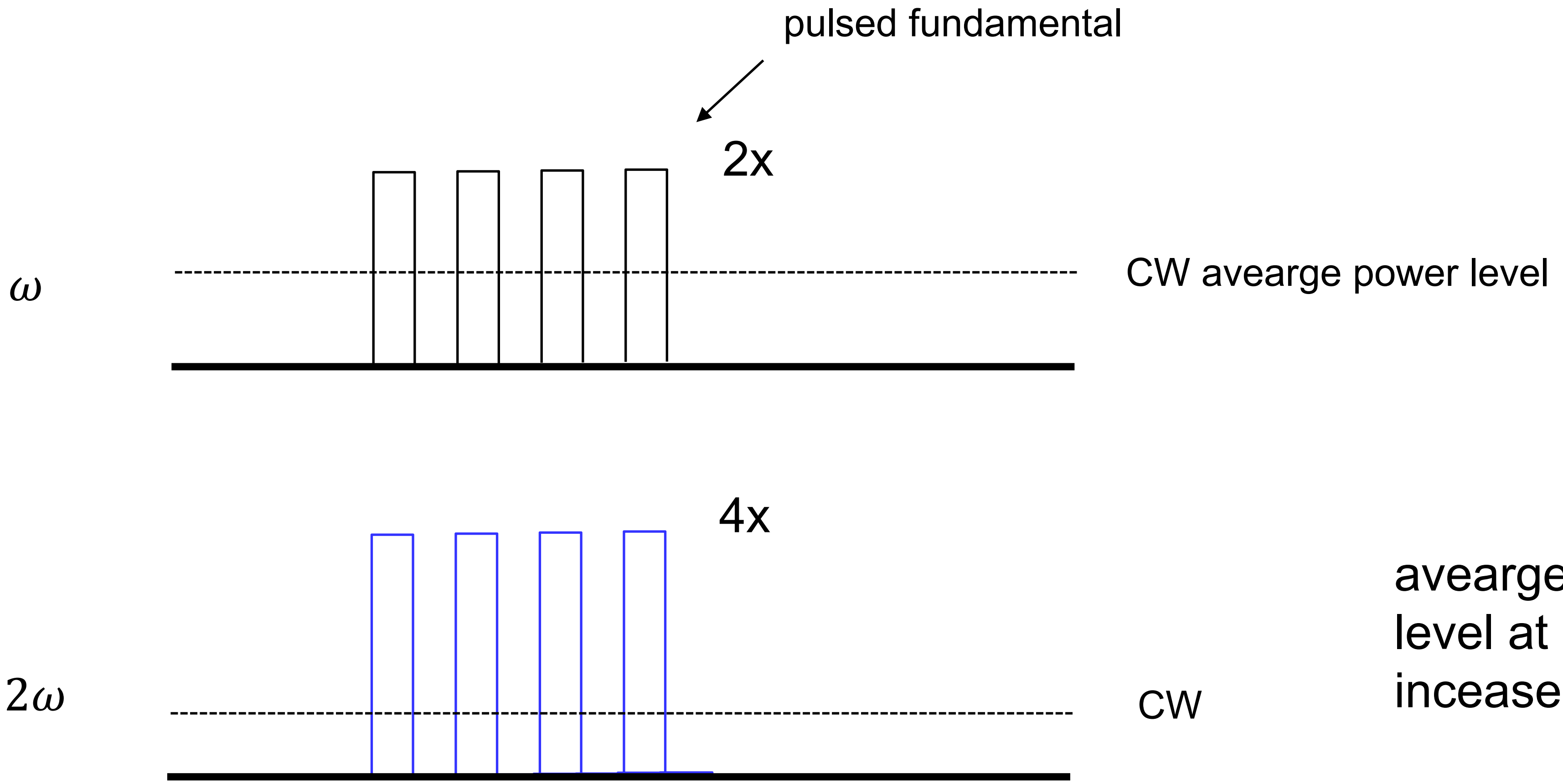


$$\eta_{NLO} \sim L^2$$

On the contary, in waveguides, the beam size is no longer limited by diffraction and is kept at a very small size (few μm) over the whole length of the crystal.

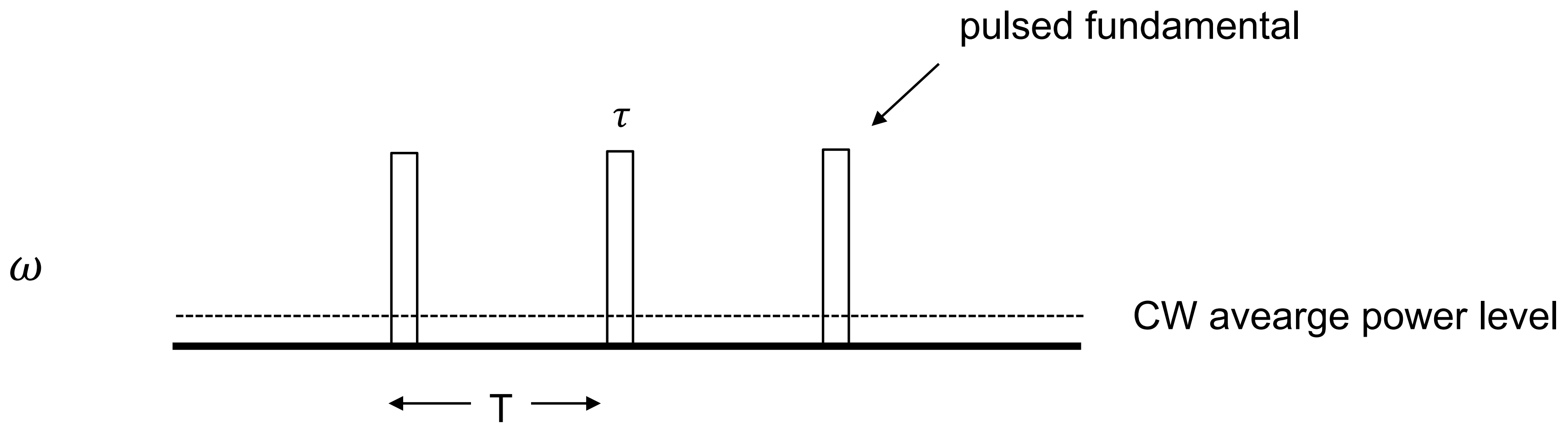
NLO conversion efficiency in waveguides can be more than 100 times higher than in bulk

SHG with pulsed radiation

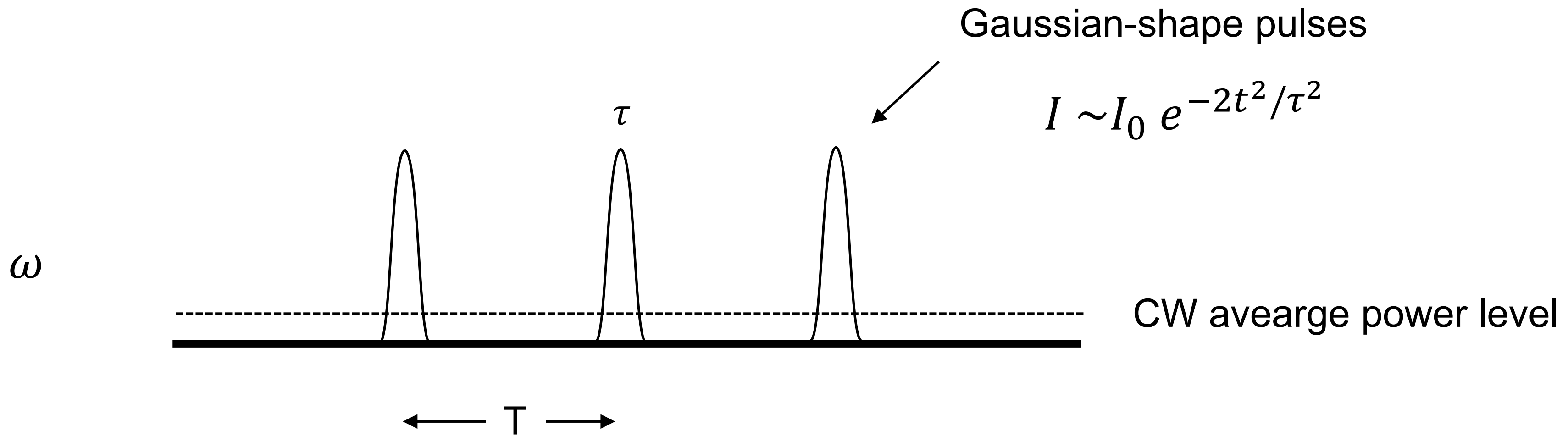


avearge power level at 2ω inceases 2^x

SHG with pulsed radiation



It is clear that at the same average power level, the SH output (and conversion efficiency) will scale as T/τ - the inverse 'duty factor'.

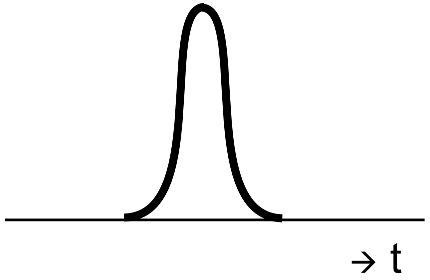


SHG, pulsed radiation Gaussian shape

Gaussian in time:

$E \sim E_0 e^{-t^2/\tau^2}$ on-axis field

$I \sim I_0 e^{-2t^2/\tau^2}$ on-axis intensity



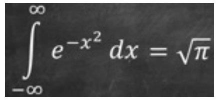
$P_\omega(t) \sim P_0 e^{-2t^2/\tau^2}$ power vs time, P_0 - peak power

$\mathcal{E}_\omega = \int_{-\infty}^{\infty} P_\omega(t)dt = \int_{-\infty}^{\infty} P_0 e^{-2t^2/\tau^2} dt$ pulse energy

For pulsed radiation we are interested in **energy conversion efficiency**

Need to **integrate power** over time

$$\mathcal{E}_{2\omega} = \int_{-\infty}^{\infty} P_{2\omega}(t)dt = \frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{L^2}{A_{eff}}\int_{-\infty}^{\infty} P_\omega^2 dt = \frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{L^2}{A_{eff}}\int_{-\infty}^{\infty} P_0^2\, e^{-4t^2/\tau^2}\, dt =$$



$$\frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{L^2}{A_{eff}} P_0^2 \int_{-\infty}^{\infty} e^{-4t^2/\tau^2}\, dt = \frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 \frac{P_0}{A_{eff}} \mathcal{E}_\omega \left\{\frac{\int_{-\infty}^{\infty} e^{-\frac{4t^2}{\tau^2}} dt}{\int_{-\infty}^{\infty} e^{-\frac{2t^2}{\tau^2}} dt}\right\} = \mathcal{E}_\omega \frac{1}{\sqrt{2}}\frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 I_0$$

$= \frac{1}{\sqrt{2}}$

$$\mathcal{E}_{2\omega} = \mathcal{E}_\omega \frac{1}{\sqrt{2}}\frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 I_0 \qquad (10.5)$$

Energy conversion efficiency

$$\mathcal{E}_{2\omega} / \mathcal{E}_\omega = \frac{1}{\sqrt{2}}\frac{\omega^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) L^2 I_0 \qquad (10.6)$$

Total reduction coeff. compared to plain wave (6.10a) because of Gaussian shape in space and time

$$\boxed{\frac{1}{2}\frac{1}{\sqrt{2}}}$$

$$I_0 \approx \frac{\mathcal{E}_\omega}{A_{eff}\tau_{eff}}$$

$$\tau_{eff} = \sqrt{\frac{\pi}{2}}\,\tau = \sqrt{\frac{\pi}{4 ln2}}\, t_{FWHM} \approx 1.064\, t_{FWHM}$$