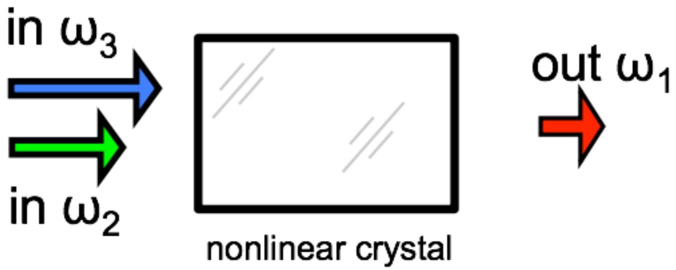
Lecture 11

Difference-frequency generation (DFG).

Difference-Frequency Generation (DFG)

(frequency down conversion)

DFG



$\omega_1 + \omega_2 = \omega_3$ SFG

$\boxed{\omega_3 - \omega_2 = \omega_1}$ DFG

ω_3, ω_2, and ω_1 are the so-called pump, signal and idler waves

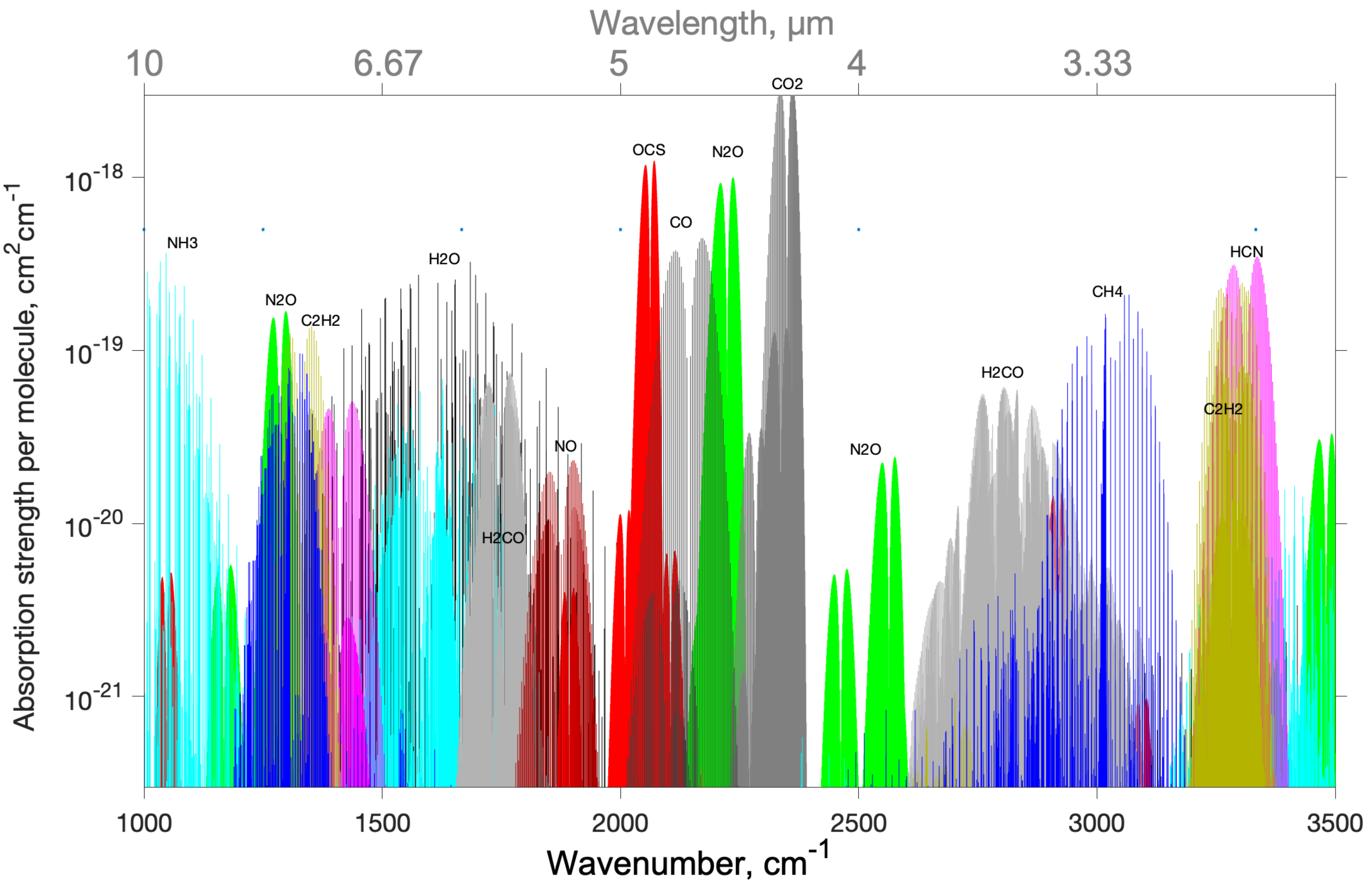
by definition $\omega_3 > \omega_2 > \omega_1$

For every photon that is created at the difference frequency ω_1, a photon at the higher input frequency (pump, ω_3) must be destroyed and a photon at the lower input frequency (signal ω_2) must be created. Hence the field at ω_2 is **amplified**.

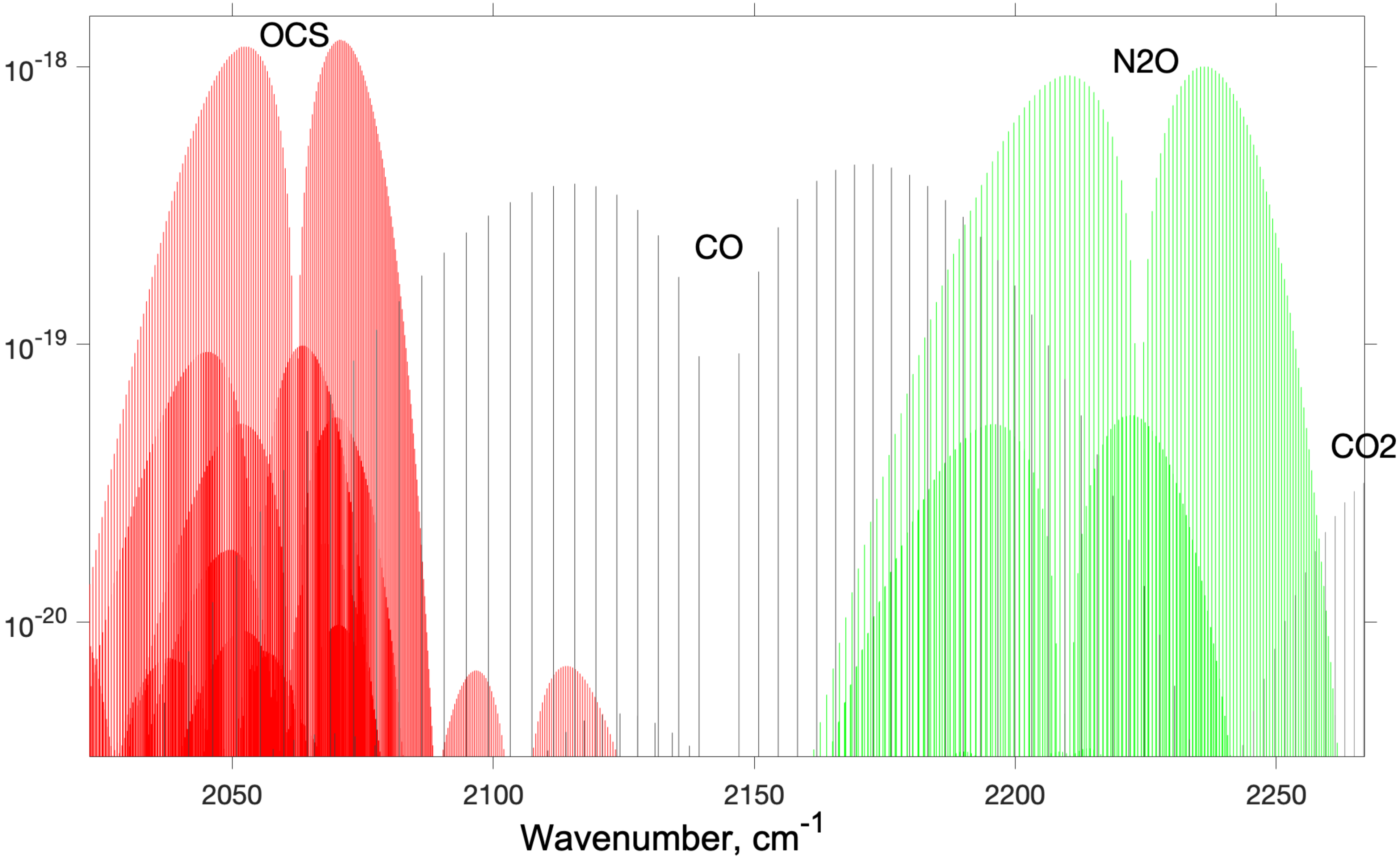
Benefits:

DFG can produce longwave IR beams (ω_1) with readily available room-temperature near-infrared 'pump' (ω_3) and 'signal' (ω_2) laser sources. For example, the outputs of telecom-range narrow-linewidth diode or fiber lasers can be fiber-coupled and mixed in a nonlinear crystal. In addition, coherence properties of the DFG output are inherited from those of the pump lasers. No threshold for this process.

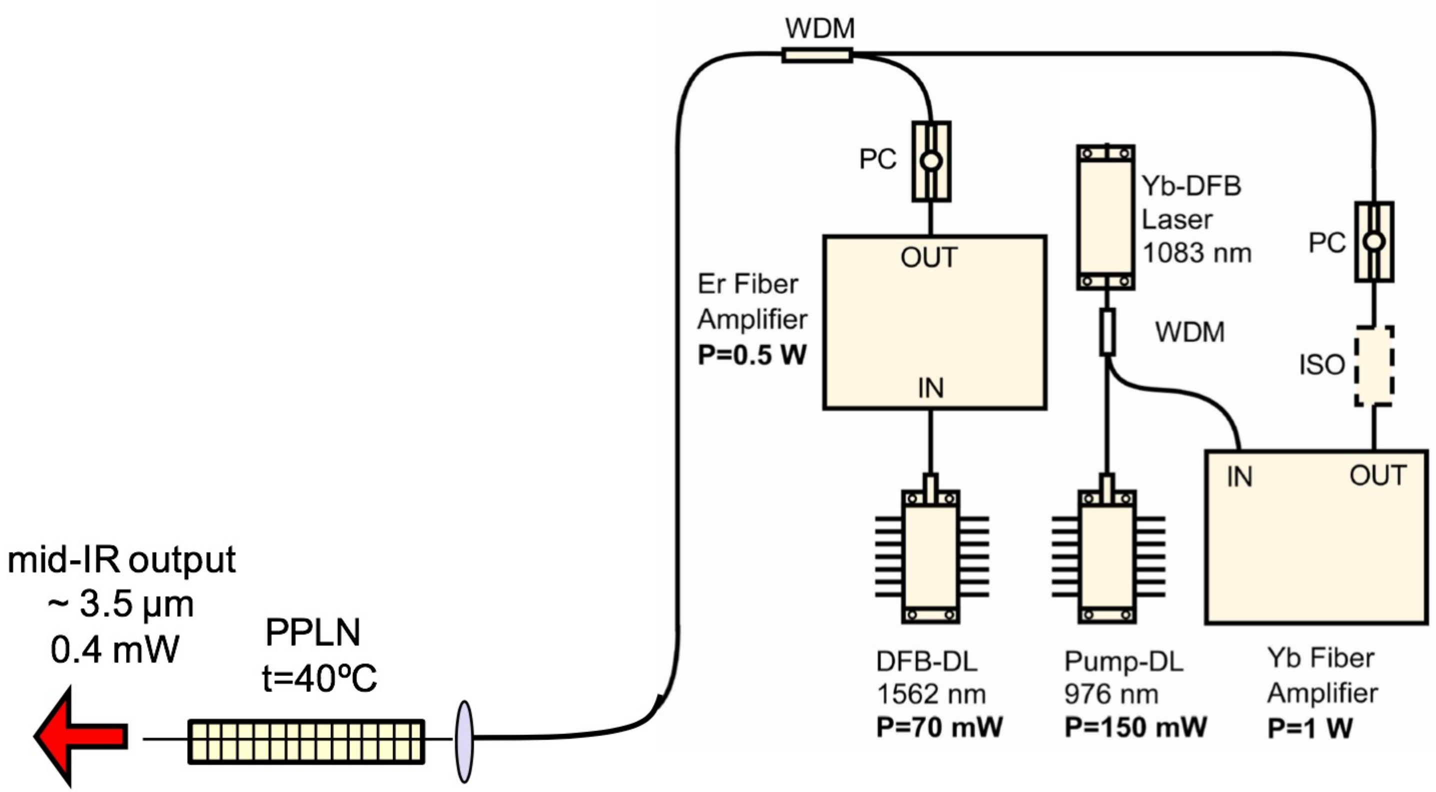
Why mid-infrared?



Why mid-infrared?



Difference-Frequency Generation
Experimental Setup

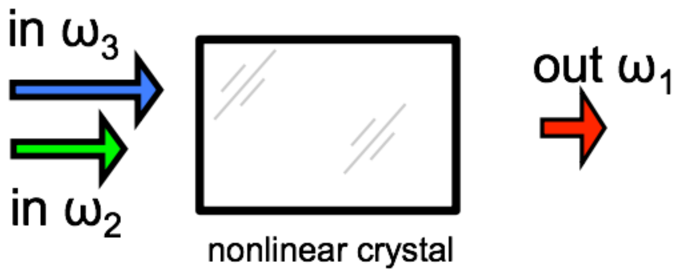


Experimental setup for the 3.5-μm DFG source based on mixing the amplified outputs of two DFB lasers in PPLN. DL, diode laser; ISO, optical isolator; PC, polarization controller; WDM, wavelength division multiplexer.

P. Weibring, D. Richter, J.G. Walega, and A. Fried, First demonstration of a high performance difference frequency spectrometer on airborne platforms, Opt. Express 15, 13476 (2007).

Difference-Frequency Generation

DFG



Assume that the **ω_3 wave and ω_2 wave are strong waves** and are undepleted by the nonlinear interaction, so that we can treat A_3 and A_2 fields as being essentially constant.

We also assume that no field is incident on the medium at frequency ω_1.

The coupled-amplitude equations describing this interaction are :

Difference-Frequency Generation (DFG)

Recall coupled-wave equations for the 3 waves (5.7)

$$\frac{dA_1}{dz} = -ig\, A_3 A_2^* e^{-i\Delta kz}$$

$$\frac{dA_2}{dz} = -ig\, A_3 A_1^* e^{-i\Delta kz}$$

$$\frac{dA_3}{dz} = -ig A_1 A_2 e^{i\Delta kz}$$

NL coupling coefficient

$$g = \frac{d}{c}\sqrt{\frac{\omega_1\omega_2\omega_3}{n_1 n_2 n_3}}$$

For A_3 and A_2 fields ≈ constant and Δk=0:

$$\frac{dA_1}{dz} = -ig\, A_3 A_2^*$$

after integration $A_1 = -ig\, A_3 A_2^* L$ – DFG field grows linearly with the length

for the normalized filed intensity $|A_1|^2$ $|A_1|^2 = g^2 |A_3|^2 |A_2|^2 L^2$

Difference-Frequency Generation

recalling that $$I_\omega = \frac{c\varepsilon_0}{2}\omega|A_\omega|^2 \quad \rightarrow \quad |A_\omega|^2 = \frac{2I_\omega}{c\varepsilon_0\omega}$$

we get $$\frac{2I_{\omega_1}}{c\varepsilon_0\omega_1} = \frac{d^2}{c^2}\frac{\omega_1\omega_2\omega_3}{n_1n_2n_3}\frac{2I_{\omega_3}}{c\varepsilon_0\omega_3}\frac{2I_{\omega_2}}{c\varepsilon_0\omega_2}L^2$$

DFG $$\boxed{I_{\omega_1} = \frac{2\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) I_{\omega_3}I_{\omega_2}L^2} \qquad (11.1)$$

recall SFG from L6: $$\boxed{I_{\omega_3} = \frac{2{\omega_3}^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right) I_{\omega_1}I_{\omega_2}L^2} \qquad (6.3)$$

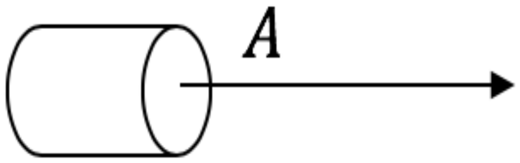
Looks similar to SFG!

But in experiment, we measure input and output **powers** → need to convert to powers

Difference-Frequency Generation

Plane-wave, near-field, top-hat approximation

Imagine, we have two input beams at ω_3 and ω_2 with average powers P_{ω_3} and P_{ω_2}



How much power P_{ω_1} at the diference frequency can we get?

Assume we have top-hat for ω_3 and ω_2 beams, with the area A. We ignore diffraction (L is small enough), so that the generated beam at ω_1 has the same area (and shape) – a near-field approximation.

Intensity (power density) at ω_3 and ω_2

$$I_{\omega_3} = P_{\omega_3}/A$$

$$I_{\omega_2} = P_{\omega_2}/A$$

Rewriting (11.1), we get:

$$\frac{P_{\omega_1}}{A} = \frac{2\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{P_{\omega_3}}{A}\frac{P_{\omega_2}}{A}L^2$$

so that

L^2 dependence

$$P_{\omega_1} = \frac{2\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{L^2}{A}P_{\omega_3}P_{\omega_2} \qquad (11.2)$$

NLO figure of merit (FOM)

smaller area improves the output

Difference-Frequency Generation (DFG)

In the limit of small conversion efficiency and perfect phase matching between the interacting waves, the DFG power P_{ω_1} is expressed by the following product:

$$P_{\omega_1} = \eta_{DFG} P_{\omega_3} P_{\omega_2}$$

Thus, P_{ω_1} is linear with respect to both P_{ω_3}, P_{ω_2}

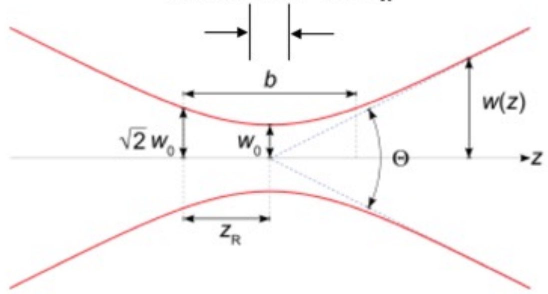
where $\eta_{DFG} = \frac{2\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{L^2}{A}$

– is a normalized conversion efficiency
in Watts per Watts^2 or Watt^{-1} or % per W
that depends on the effective nonlinear coefficient (d_{eff}), the output mid-IR frequency (ω_1), focusing strength, and interaction length (*L*).

One can also see that the DFG conversion efficiency drastically drops with increasing the mid-IR wavelength due to the $\omega_1^2 \sim 1/\lambda_1^2$ term

Difference-Frequency Generation, Gaussian beams

near field $z << z_R$



Let us now calculate (Similar to Lecture 10) DFG **power** conversion efficiency for Gaussian beams in the near-field approximation and low conversin limit.

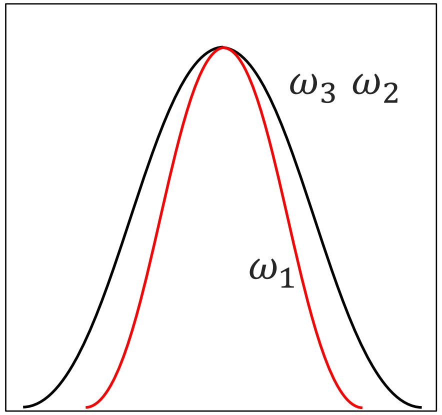
$$\boxed{E \sim e^{-r^2/w_0^2}, \;\; I \sim e^{-2r^2/w_0^2}}$$

Assume that the ω_3 and ω_2 beams have the same beamsize w_0 (1/e$_2$ intensity beam radius).

The DFG beam at ω_1 has the beamsize $w_0/\sqrt{2}$

$$I \sim e^{-4r^2/w_0^2}$$

Need to integrate over XY plane.



Difference-Frequency Generation, Gaussian beams

From f-la (11.1):

On-axis (max) DFG intensity
$$I_{\omega_1 0} = \frac{2\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)L^2 \times I_{\omega_3 0} I_{\omega_2 0} \qquad (11.3)$$

Integrated DFG intensity = power
$$P_{\omega_1} = \int I_{\omega_1} dxdy = \int I_{\omega_1 0}\; e^{-4r^2/w_0^2} 2\pi r dr = I_{\omega_1 0}\frac{\pi w_0^2}{4}$$

$A_{eff}/2$

At the same time (for Gauss):
$$P_{\omega_3} = I_{\omega_3 0}\frac{\pi w_0^2}{2} = I_{\omega_3 0}A_{eff}$$
and
$$P_{\omega_2} = I_{\omega_2 0}\frac{\pi w_0^2}{2} = I_{\omega_2 0}A_{eff}$$

Thus it follows from (11.3) that
$$\frac{P_{\omega_1}}{A_{eff}/2} = \frac{2\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)L^2 \times \frac{P_{\omega_3}}{A_{eff}}\frac{P_{\omega_2}}{A_{eff}}$$

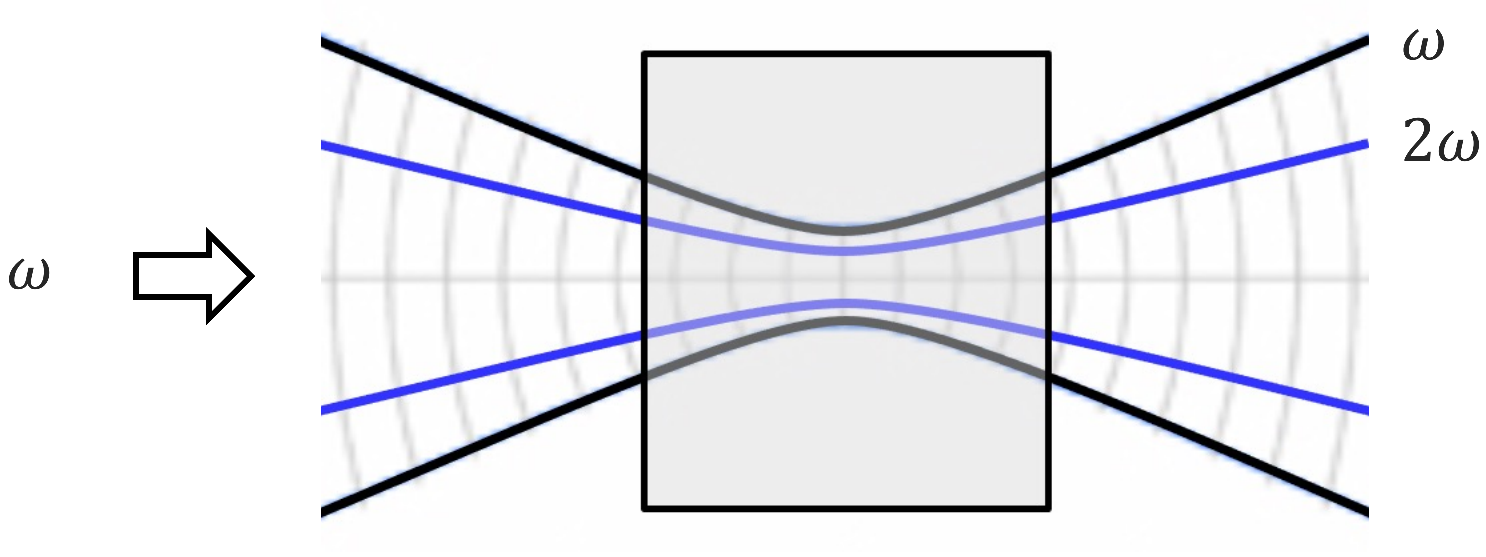
and finally
$$\boxed{P_{\omega_1} = \frac{2\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{1}{2}\frac{L^2}{A_{eff}}P_{\omega_3}P_{\omega_2} = \frac{\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{L^2}{A_{eff}}P_{\omega_3}P_{\omega_2}} \qquad (11.4)$$

factor $\frac{1}{2}$ due to Gaussian shape aveaging

effective pump beam area

Comparison:

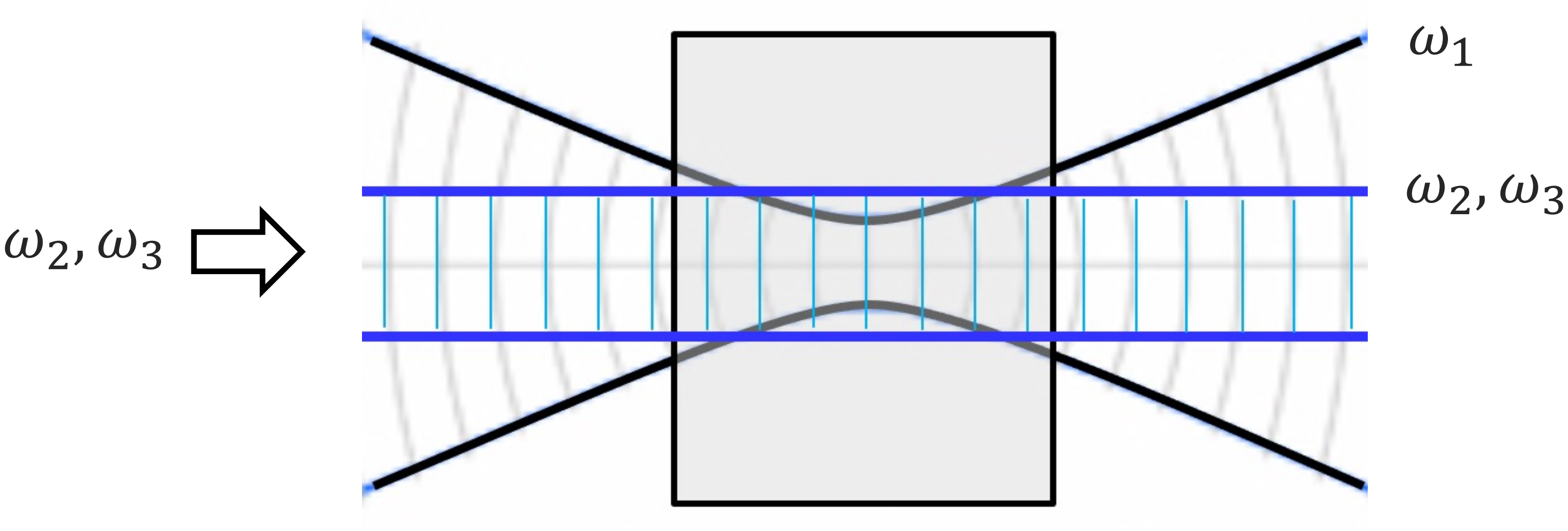
1) SHG with focused beams : $\omega \rightarrow 2\omega$



The Gaussian fundamental beam (solid line) and its 2-nd harmonic (dashed line) propagate in a similar way (and have the same Rayleigh length z_R). The radii of curvature coincide since

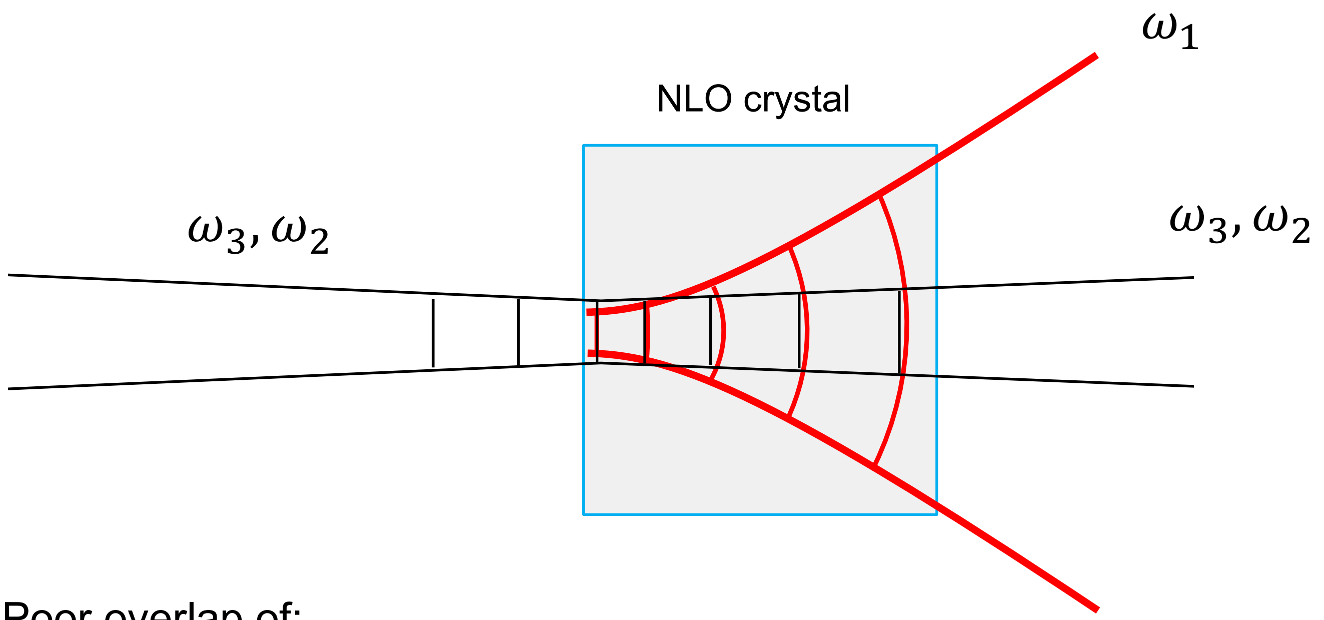
$$R(z) = z\left[1+\left(\frac{z_R}{z}\right)^2\right]$$

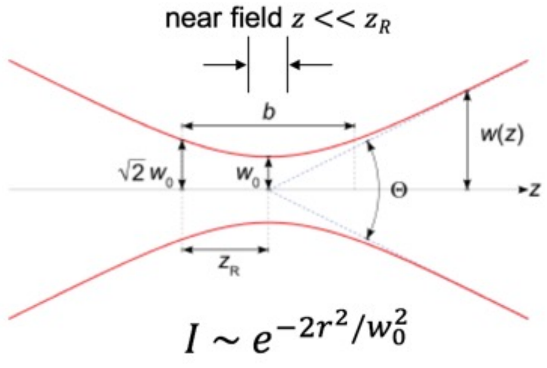
2) Difference-Frequency Generation with focused beams



The wavefront radii of curvature are different

Difference-Frequency Generation





Poor overlap of:
- beam waist radii w
- wave fronts

→ inability of tight focusing

twice larger for IR beam

$$z_R = \frac{\pi w_0^2}{\lambda / n}$$

~ can be 5 times
larger for IR beam

Because of the poor overlap, one can not focus the pump tightly – need to have the pump beams to be such that for the geneарted IR beam (ω_1): z_R> crystal length.

For IR beam: z_R> crystal length for ω_3, ω_2 and z_R< crystal length

Difference-Frequency Generation

In quantum mechanics (QM), the interaction Hamiltonian is proportional to the overlap integral.

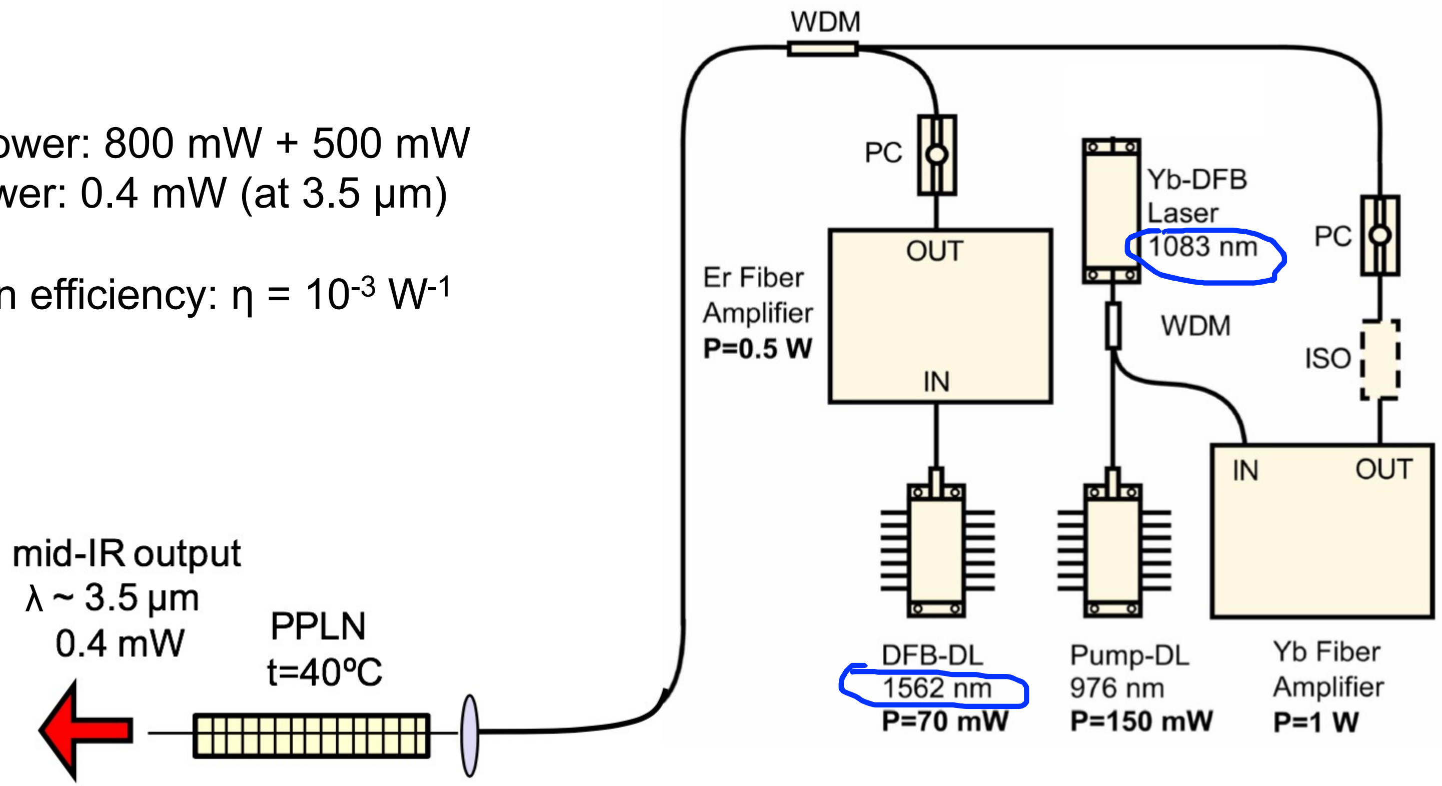
It is the **overlap integral**, which is responsible for **phase matching** in QM.

$$\hat{H}_{NL} \sim \chi^{(2)} \boxed{\int E_1^*(r) E_2^*(r) E_3(r)\, d^3 r}$$

Gaussian beams, DFG, numerical examples

Near-IR power: 800 mW + 500 mW
Mid-IR power: 0.4 mW (at 3.5 μm)

Conversion efficiency: $\eta = 10^{-3}\ W^{-1}$

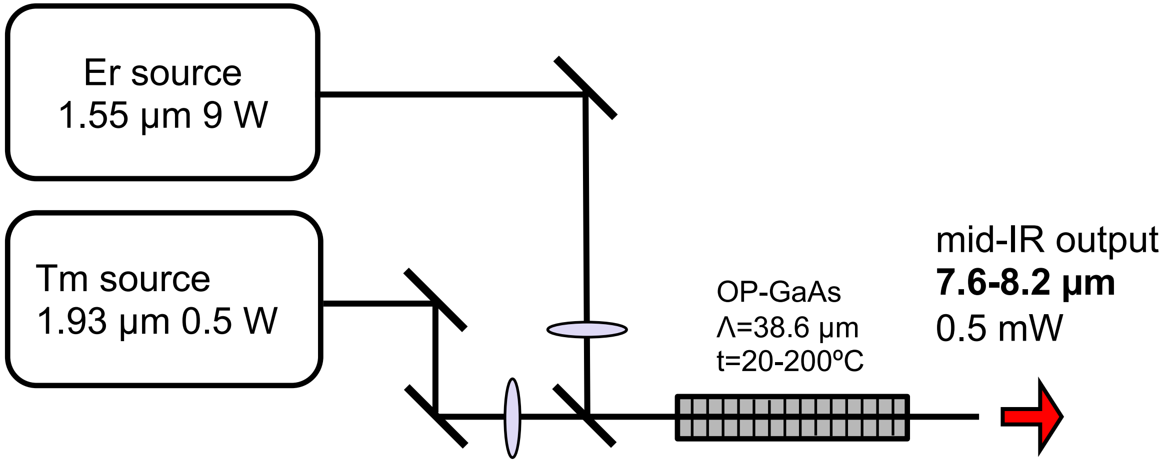


Experimental setup for the 3.5-μm DFG source based on mixing the amplified outputs of two DFB lasers in PPLN. DL, diode laser; ISO, optical isolator; PC, polarization controller; WDM, wavelength division multiplexer.

P. Weibring, D. Richter, J.G. Walega, and A. Fried, First demonstration of a high performance difference frequency spectrometer on airborne platforms, Opt. Express 15, 13476 (2007).

DFG in orientation-patterned GaAs (OP-GaAs)

$$P_{out} = \eta \times P_1 P_2$$



Near-IR power: 9 W + 0.5 W
Mid-IR power: 0.5 mW
Conversion efficiency: ~ 10^{-4} W^{-1}

S. Vasilyev, S. Schiller, A. Nevsky, A. Grisard, D. Faye, E. Lallier, Z. Zhang, A. J. Boyland, J. K. Sahu, M. Ibsen, and W. A. Clarkson, "Broadly tunable single-frequency cw mid-infrared source with milliwatt-level output based on difference-frequency generation in orientation-patterned GaAs," Opt. Lett. **33**, 1413 (2008).

Gaussian beams, DFG, numerical examples

$$P_{\omega_1} = \frac{\omega_1^2}{\varepsilon_0 c^3} \left(\frac{d^2}{n^3}\right) \frac{L^2}{A_{eff}} \; P_{\omega_3} P_{\omega_2}$$

see (11.4)

PPLN crystal

λ_3 =1083nm
λ_2=1562nm
λ_1=3.53 µm ω_1=5.34e14 s^{-1}

ϵ_0=8.85e-12 F/m
c=3e8 m/s

d_{33}=22 pm/V, (for mid-IR)
eee quasi phase matching,
$d_{eff}=\frac{2}{\pi}d_{33}$ =14 pm/V=14e-12 m/V
n=2.14;
L=5 cm (5e-2 m)
w_0=230 µm (Gauss), $A_{eff}=\frac{\pi w_0^2}{2}$ =8.3e-8 m^2
$\boldsymbol{P_{\omega_3} = P_{\omega_2}}$**=1 W**

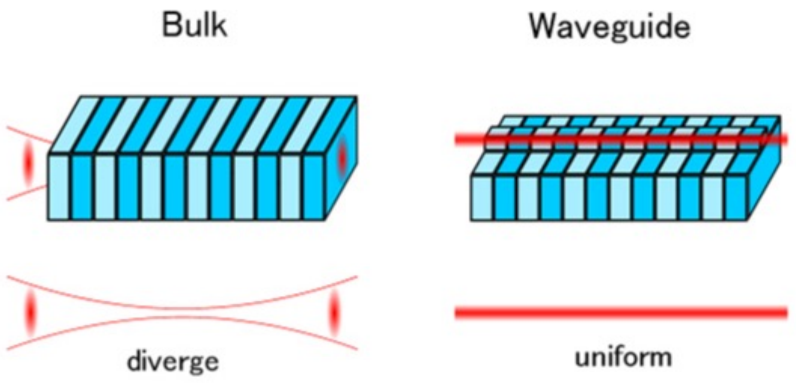
P_{ω_1}= 5.34e14^2/8.85e-12/3e8^3 *(14e-12^2/2.14^3) *5e-2^2/8.3e-8 *1^2 =

= 7.2e-4 W = 0.72 mW

Power conversion efficiency

$$\eta_{DFG} = \frac{0.72mW}{1W \cdot 1W} = 0.72 \cdot 10^{-3} \; W^{-1}$$

DFG in PPLN waveguide ~ 100 times more efficient



Zn:LiNbO3 ridge **waveguide** fabricated by direct bonding (11 μm thick, 17 μm wide and 38 mm long)

1.064 μm 'pump' generated with a laser diode and amplified in an ytterbium-doped fiber amplifier
+
1.55 μm 'signal' generated by an external cavity laser diode and an erbium-doped fiber amplifier.

Near-IR power: 444 mW + 558 mW
Mid-IR power: **65 mW** at 3.4 μm
Conversion efficiency: ~ 0.26 W^{-1}

M. Asobe, O. Tadanaga, T. Yanagawa, T. Umeki, Y. Nishida, and H. Suzuki, High-power mid-infrared wavelength generation using difference frequency generation in damage-resistant Zn: LiNbO3 waveguide, Electron. Lett. 44, 288 (2008).

DFG in a waveguide, numerical example

For WG, we use formula (11.2)

$$P_{\omega_1} = \frac{2\omega_1^2}{\varepsilon_0 c^3}\left(\frac{d^2}{n^3}\right)\frac{L^2}{A_{eff}} P_{\omega_3} P_{\omega_2}$$

see (11.2)

PPLN crystal

λ_3 =1064nm
λ_2=1550nm
λ_1=3.4 μm ω_1=5.54e14 s^{-1}

ϵ_0=8.85e-12 F/m
c=3e8 m/s

d_{33}=22 pm/V,
eee quasi phase matching,
$d_{eff}=\frac{2}{\pi}d_{33}$ =14 pm/V
n=2.14;
L=38 mm
$A_{eff}= 11x17\ \mu m^2 = 1.9$e-10 m^2
$P_{\omega_3}= 0.444\ W, P_{\omega_2}=0.558\ W$

P_{ω_1}= 2*5.54e14^2/8.85e-12/3e8^3 *(14e-12^2/2.14^3) *3.8e-2^2/1.9e-10 *0.444*0.558 =

= 0.097 W ~ 100 mW

Power conversion efficiency

$$\eta_{DFG} = \frac{97mW}{444mW \cdot 558mW} = 0.4\ W^{-1}$$

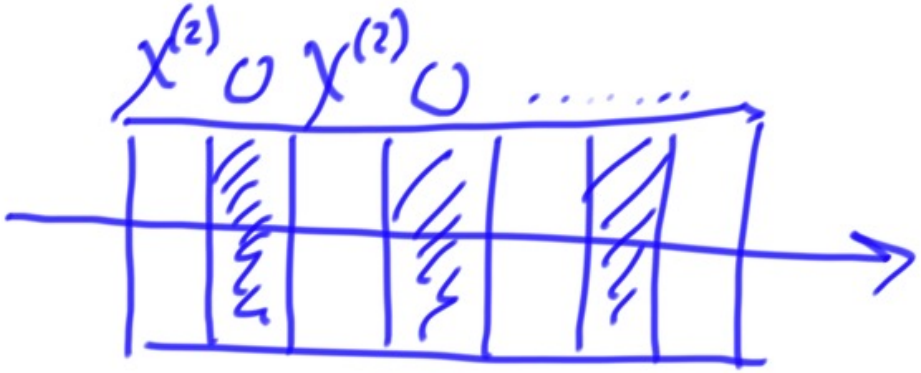
Common nonlinear optical crystals used for DFG

Crystal	Transparency range (μm)	d_{eff} (pm/V)	Average ref. index	NLO FOM d_{eff}^2/n^3 with respect to PPLN
Periodically poled oxides				
PP LN	0.4-5.5	$2/\pi \cdot 22.3=14.2$	2.13	1
PP KTP	0.35-4.3	$2/\pi \cdot 16.9=10.8$	1.8	0.95
Birefringent				
AGS	0.47-11	12	2.4	0.5
AGSe	0.71-19	33	2.65	2.8
ZGP	1-12	75	3.13	8.8
GaSe	0.62-20	54	2.73	6.8
Orientation patterned				
OP-GaAs	0.9-17	$2/\pi \cdot 94=60$	3.3	6.2
OP-GaP	0.5-12	$2/\pi \cdot 37=23.6$	3.05	2.30

Difference-Frequency Generation

Other ways of phase matching in DFG :

1. Phase matching via zeroing NLO coefficient every half period



2. Phase matching via modal dispersion e.g. TE and TM