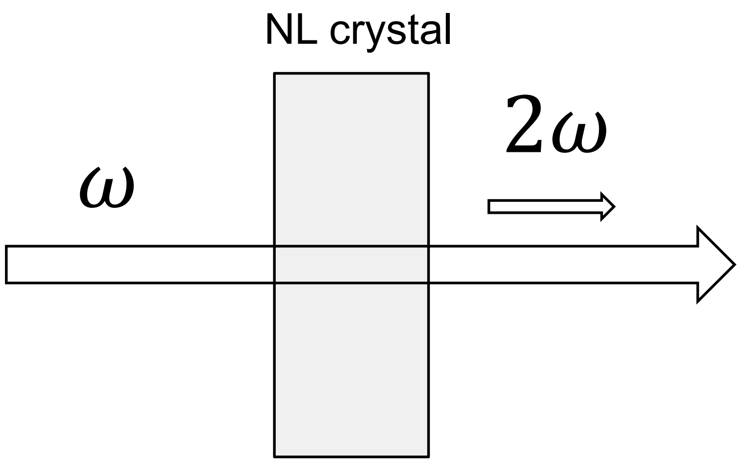
Lecture 14

Harmonics generation inside a resonant cavity.

Second harmonic generation

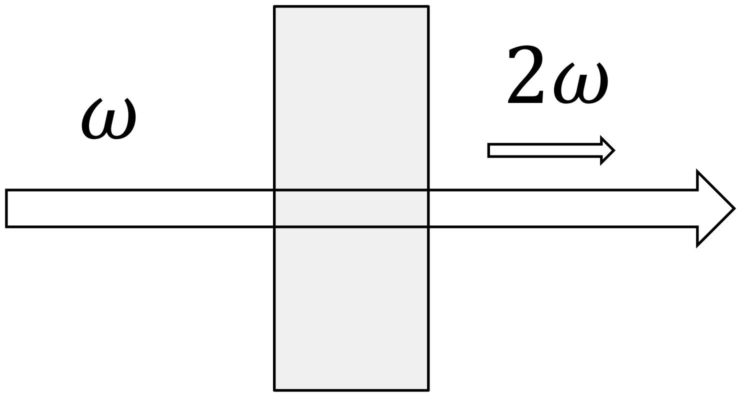
Second harmonic generation is instrinsically a **weak process**. In the first observations of SHG by Franken et al., 2ω photons (in the UV) were hardly detectable. They used a pulsed ruby laser and a crystalline quartz as nonlinear crystal, where the chromatic dispersion limited the interaction length to about 14 microns.



P. A. Franken, A.-E. Hill, C. W. Peters, and G. Weinreich,
“Generation of optical harmonics, Phys. Rev. Lett., vol. 7, pp. 118-119. August 1961.

Second harmonic generation

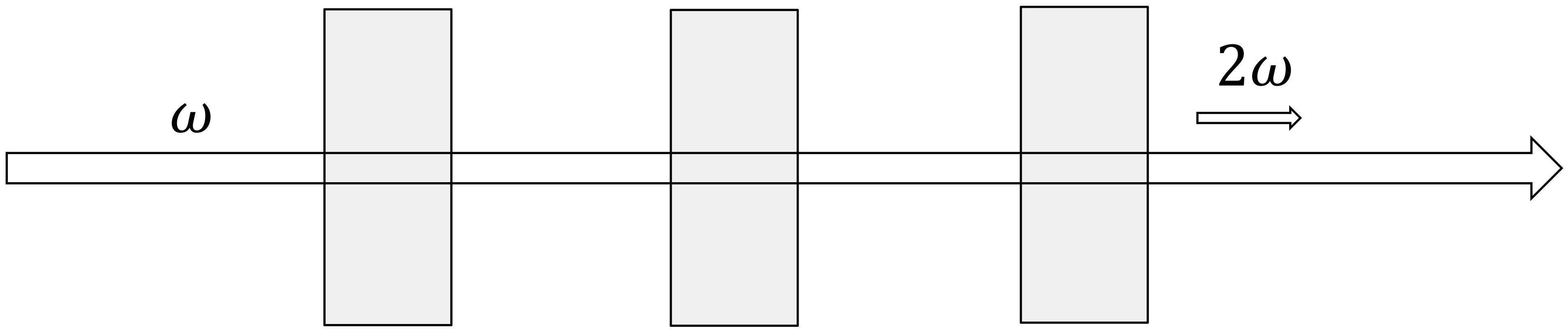
How to improve SHG efficiency at a given pump laser power ?



- Tighter focusing?
- Longer crystals? 5cm? 10 cm?
- Shorter pulses?
- Crystals with higher nonlinearity d_{eff} ?
- Waveguides?

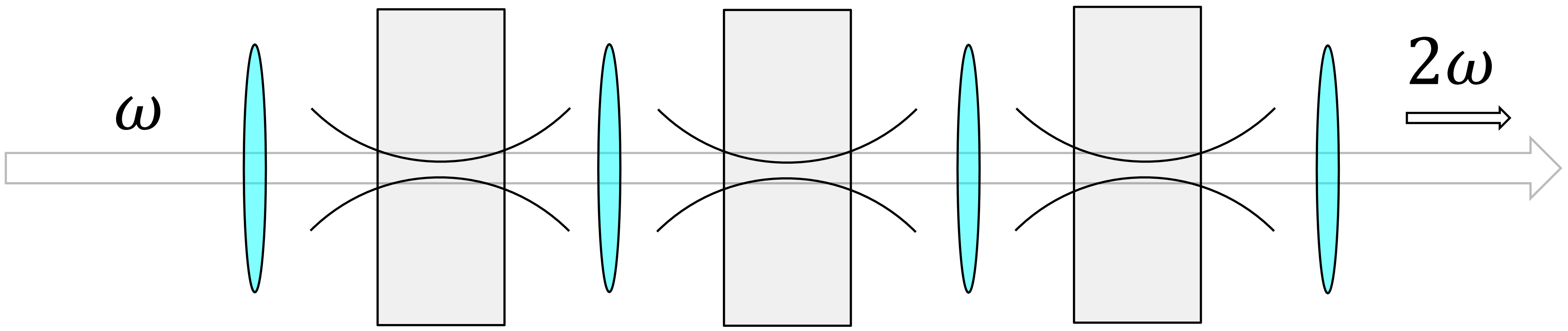
Second harmonic generation

Put many crystals in a sequence?



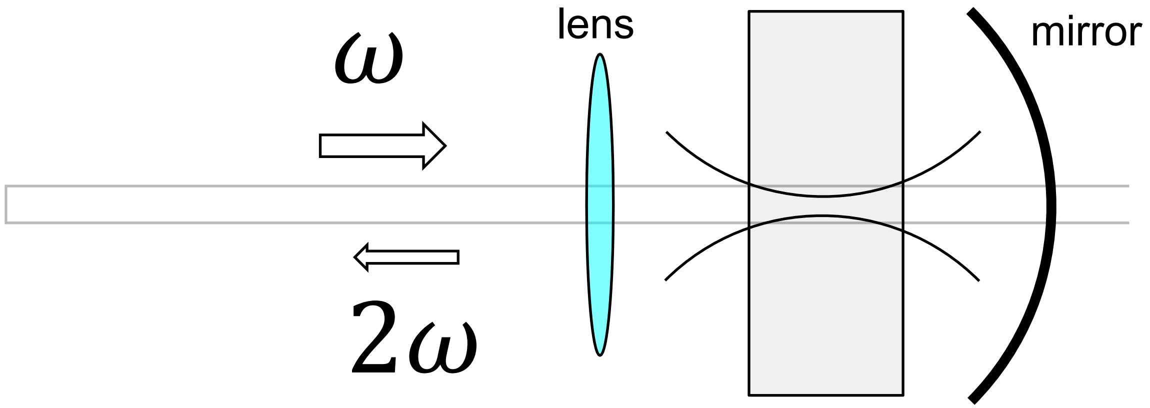
Second harmonic generation

Put many crystals in a sequence with periodic refocusing

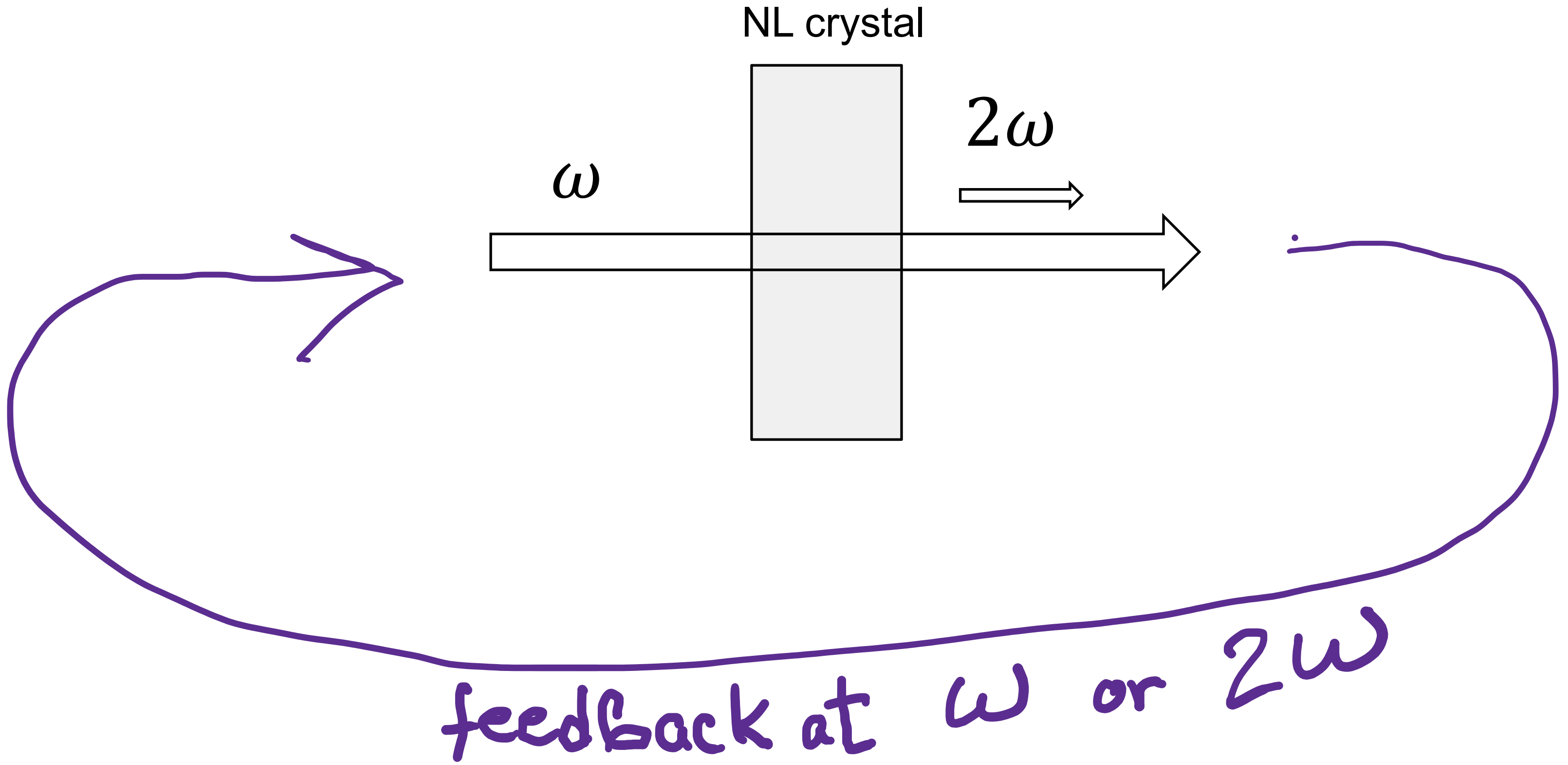


Second harmonic generation

Put a retroreflector for a double pass?



Second harmonic generation with a feedback



J. A. Armstrong, N. Bloembergen, J. Ducuing, and P. S. Pershan, “Interactions between light waves in a nonlinear dielectric”, Phys. Rev., vol. 127, pp. 1918 (1962).

Resonant Optical Second Harmonic Generation

Nobel prize in Physics 2018

IEEE JOURNAL OF QUANTUM ELECTRONICS VOL. QE-2, NO. 6 JUNE 1966

Resonant Optical Second Harmonic Generation and Mixing

A. ASHKIN, MEMBER, IEEE, G. D. BOYD, MEMBER, IEEE, AND J. M. DZIEDZIC, MEMBER, IEEE

Abstract—Experimental and theoretical results are described on the enhancement of optical second harmonic generation (SHG) and mixing in KDP by the use of optical resonance. Both resonance of the harmonic and of the fundamental are considered. Large enhancements are possible for resonators with low loss. Using a plano-concave harmonic resonator containing 1.23 cm of KDP, the authors achieved a loss < 4 percent per pass. This resulted in an enhancement of ~ 500 times the harmonic power internal to the resonator and ~ 10 times external to the resonator. When resonating, the fundamental enhancements of ~ 5 were observed.

The theory includes the effect of double refraction. This results in a coupling coefficient of the generated harmonic power to the transverse modes of the harmonic resonator. The experimental results are in substantial agreement with the theory.

I. Introduction

THIS PAPER DEALS with the use of optical resonance to enhance the efficiency of optical

It was found that the gas laser, because of its single transverse mode [12], [13] capability, could produce considerable SHG power on a CW basis, in spite of its relatively low power. Working in the lowest-order transverse mode of a 3.4 mm diameter beam close to the beam minimum permitted observation of interaction over the full length of a 1.2 cm-long KDP crystal, and led to a new determination of the nonlinear coefficient [9] of KDP. Further work [7], [8] on crystals as long as 5 cm and 10 cm showed, both experimentally and theoretically, how double refraction limits the interaction length in essentially parallel beams of finite transverse extent. Due to the physical separation of the ordinary wave fundamental beam from the second harmonic extraordinary wave beam, efficient interaction can be maintained up to a length l_a, called the aperture length [7], where

Resonant Optical Second Harmonic Generation

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.... Experimental and theoretical results are described on the enhancement of optical second harmonic generation (SHG) and mixing in KDP by the use of optical resonance. Both resonance of the **harmonic** and of the **fundamental** are considered.

... Large enhancements are possible for resonators with low loss. Using a plano-concave **harmonic** resonator containing 1.23-cm KDP, the authors achieved a loss < 4 percent per pass. This resulted in an enhancement of ~ 500 times the harmonic power **internal** to the resonator and ~ 10 times external to the resonator.

...When resonating, the **fundamental** enhancements of ~ 5 were observed.

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Resonant Optical Second Harmonic Generation

IEEE JOURNAL OF QUANTUM ELECTRONICS VOL. QE-2, NO. 6 JUNE 1966

Resonant Optical Second Harmonic Generation and Mixing

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Used a low power (mW’s) continuous-wave (CW) HeNe laser operating at λ=1.15 μm

Idea #1

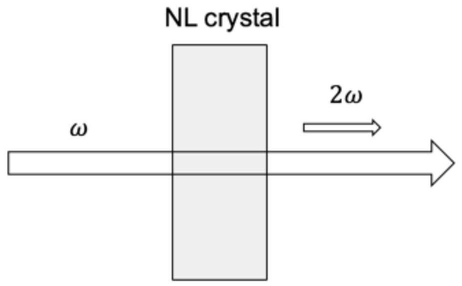
... optical feedback or resonance can be used effectively to extend the interaction length and enhance the amount of harmonic conversion. ... the output **harmonic** beam is reflected back and refocused by an optical resonator in such a phase that it can continue to interact with the fundamental power. In t'his way the effective length of interaction can be increased greatly...

Idea #2

.... resonating the incident **fundamental** beam. This is simply a means of storing the incident energy in a high Q cavity to increase the flux of fundamental power passing through the crystal, thereby increasing the generation of second harmonic power...

Resonant Optical Second Harmonic Generation

Idea #1



Recall (6.9) from Lecture 6

second harmonic fundamental

$$\frac{dA_3}{dz} = -i\frac{g}{2}A_1{}^2$$

$$A_i = \sqrt{\frac{n_i}{\omega_i}}E_i$$

$$g = \frac{d}{c}\sqrt{\frac{\omega_1\omega_2\omega_3}{n_1n_2n_3}}$$

For single pass and low conversion limit (A_1≈const), the SH field increases by $\Delta A_3 = A_3 = -i\frac{g}{2}A_1{}^2 L$

Thus the SH power generated in a single pass is $\Delta P_{2\omega} = P_{2\omega} \sim |A_3|^2 = (\frac{g}{2})^2|A_1|^4L^2$

Now imagine we have a SH input field A_{30}; the SH field increases by the same amount: $\Delta A_3 = -i\frac{g}{2}A_1{}^2 L$

What is an extra SH power generated in a single pass with non-zero input?

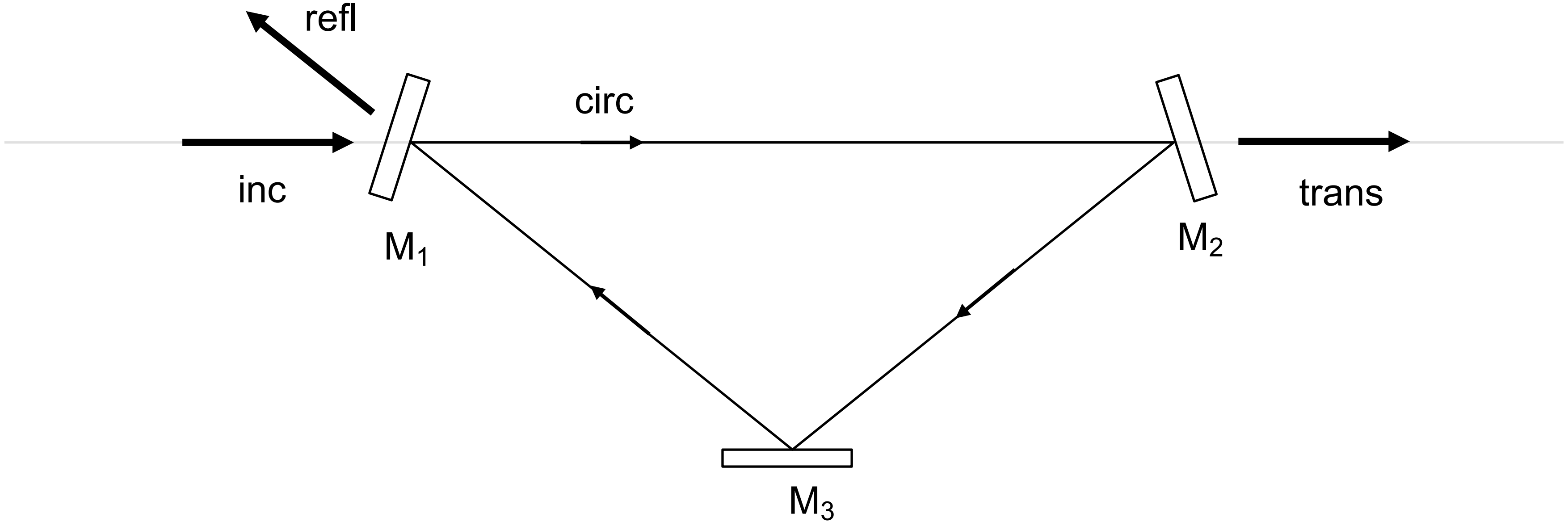
$$\Delta P_{2\omega} \sim \Delta|A_3|^2 = \underbrace{2A_{30}\Delta A_3} \gg |\Delta A_3|^2$$

if the input SH field is already large enough, $A_{30} > \Delta A_3$

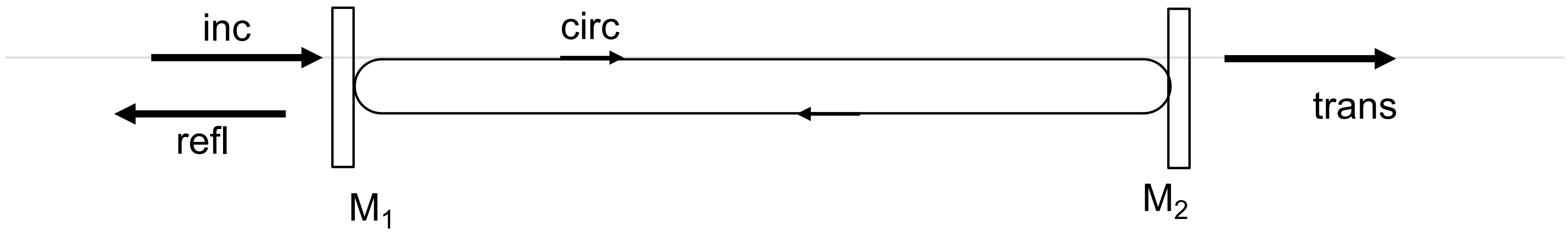
Resonant properties of passive optical cavities

(after Siegman, *Lasers*)

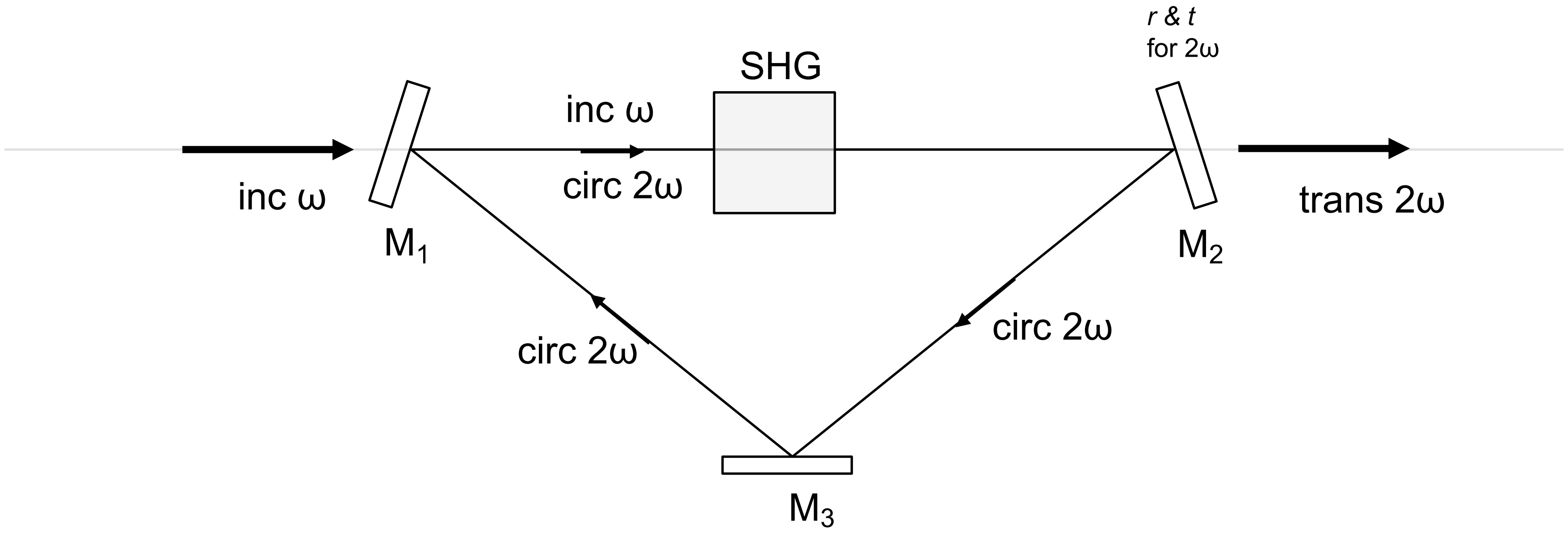
Ring



Planar



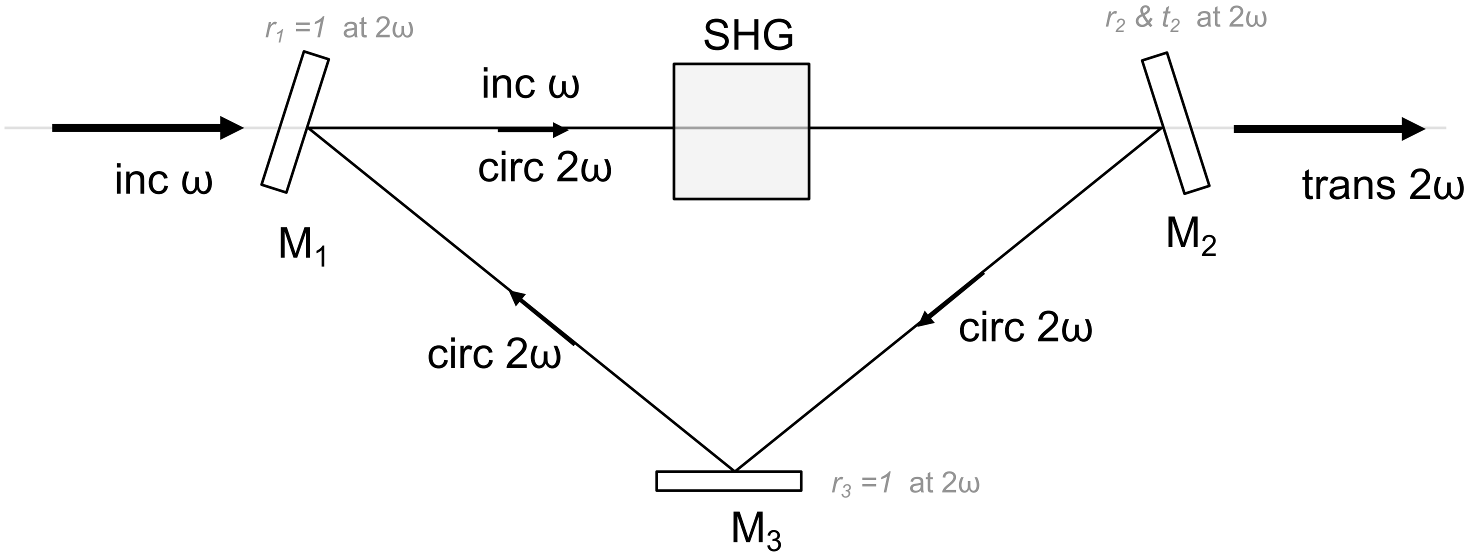
Resonanting Second Harmonic



Ideal scenario:

M1 - transmits 100% of the pump (ω)
M1 & M3 - reflect 100% of the SH (2ω)
M2 - highly reflects 2ω with the *field* reflection coeff. *r* and *field* transm. coeff. *t*

Resonating Second Harmonic



$$A_i = \sqrt{\frac{n_i}{\omega_i}} E_i$$

$$\Delta A_3 \sim \Delta E_3$$

Write balance equation for a roundtrip (self-consistency equation) for the *E*- field at 2ω :

$$\alpha_E E_{circ,2\omega} + (1 - r_2) E_{circ,2\omega} = \Delta E_{2\omega}$$

all other roundtrip losses in the cavity (α_E<<1)

reflection roundtrip loss (<<1)

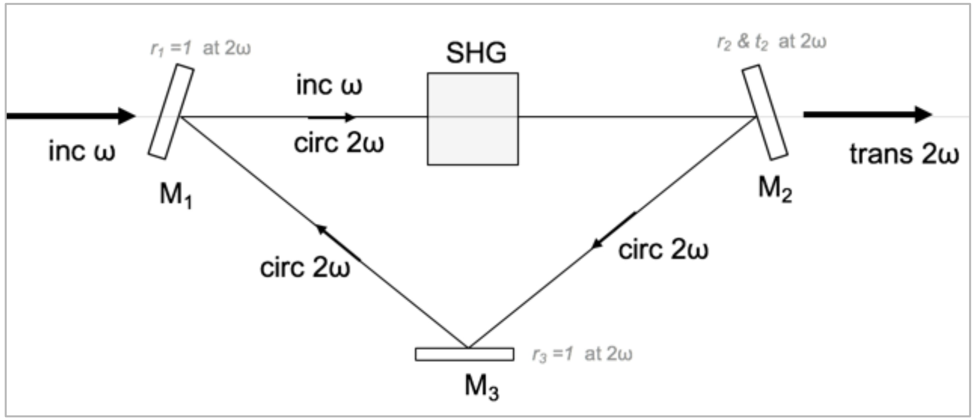
2ω field added due to SHG

$r_2^2 + t_2^2$=1 in *E*-field

$R_2 + T_2 = 1$ in intensity (or power)

→ $$E_{circ,2\omega} = \frac{\Delta E_{2\omega}}{\underbrace{1 - r_2}_{} + \alpha_E} \qquad (14.1)$$ circulating SH *E*-field

Resonating Second Harmonic



circulating SH *power*

$$P_{circ,2\omega} = \frac{|\Delta E_{2\omega}|^2}{(1 - r_2 + \alpha_E)^2} = \frac{P_{single-pass,2\omega}}{(1 - r_2 + \alpha_E)^2} \qquad (14.2)$$

α_E=$\alpha_{E,2\omega}$

( ) can be on the order of few % →
intracavity SHG enhancement > 1000

outcoupled (transmitted) SH *power*

$$P_{out,2\omega} = \frac{t_2^2}{(1 - r_2 + \alpha_E)^2} P_{single-pass,2\omega} \qquad (14.3)$$

ideal mirror:

$r_2^2 + t_2^2$ =1 in *E*-field

$R_2 + T_2 = 1$ in intensity (power)

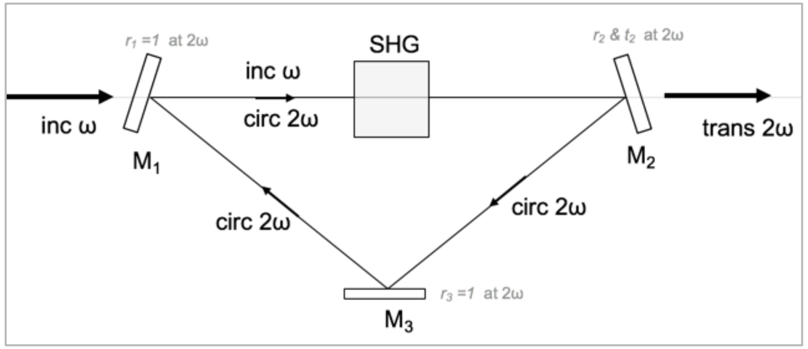
SHG output enhancement factor

$$Enh\_factor = \frac{P_{out,2\omega}}{P_{single-pass,2\omega}} = \frac{t_2^2}{(1 - r_2 + \alpha_E)^2} =$$

$$= \frac{1 - r_2^2}{(1 - r_2 + \alpha_E)^2} \approx \frac{2(1 - r_2)}{(1 - r_2 + \alpha_E)^2} \qquad (14.4)$$

because r_2 is close to 1, $r_2 + 1 \approx 2$

Resonating Second Harmonic



$1 - r_2 = x$

When other losses are zero: α_E=0, $Enh.factor$ can be any:

$$Enh\_factor = \frac{P_{out,2\omega}}{P_{single-pass,2\omega}} = \frac{2(1-r_2)}{(1-r_2+\alpha_E)^2} = \frac{2x}{(x+\alpha_E)^2}$$ - find maximum!

derivative: $$\frac{2}{(x+\alpha_E)^2} - \frac{2\times 2x}{(x+\alpha_E)^3} = 0;\;\; x+\alpha_E = 2x;\;\; x = \alpha_E;\;\; 1-r_2 = \alpha_E$$

$Enh.factor$ is maximized when outcoupling loss is equal to other losses: $1 - r_2 = \alpha_E$

so-called 'impedance matching'

$$Enh\_factor = \frac{P_{out,2\omega}}{P_{single-pass,2\omega}} \approx \frac{2(1-r_2)}{(1-r_2+\alpha_E)^2} = \frac{2\alpha_E}{(2\alpha_E)^2} = \frac{1}{2\alpha_E} = \frac{1}{\alpha_I} \qquad (14.5)$$

roundtrip passive loss in the cavity, e.g. loss in the crystal or diel. mirrors

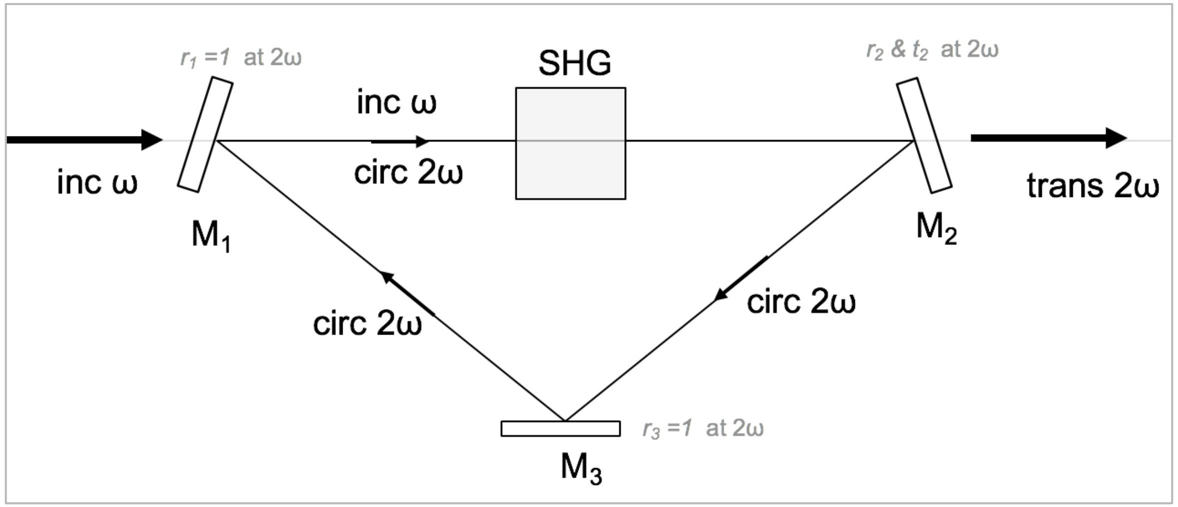
α_E- field loss

α_I- power (intensity) loss

roundtrip power loss

Example: α_I=1% gives you 100 times higher SHG eficiency

Resonating Second Harmonic



Of course, the formula we used for the single pass SHG

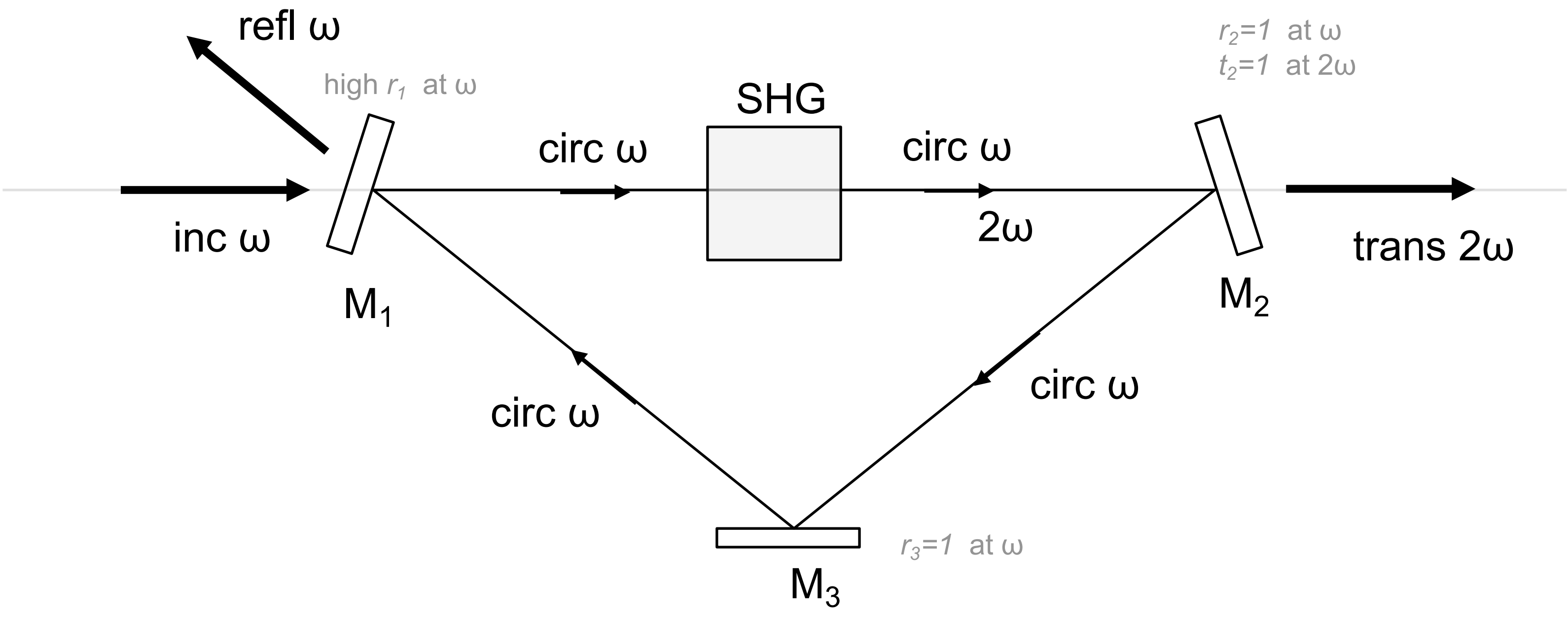
$$\frac{dA_3}{dz} = -i\frac{g}{2}{A_1}^2$$

is for low conversion limit: $A_3 \approx const$

However, the 2ω power inside the cavity can be much higher than the pump power at ω !
The gain factor will eventually saturate.

Resonating Fundamental

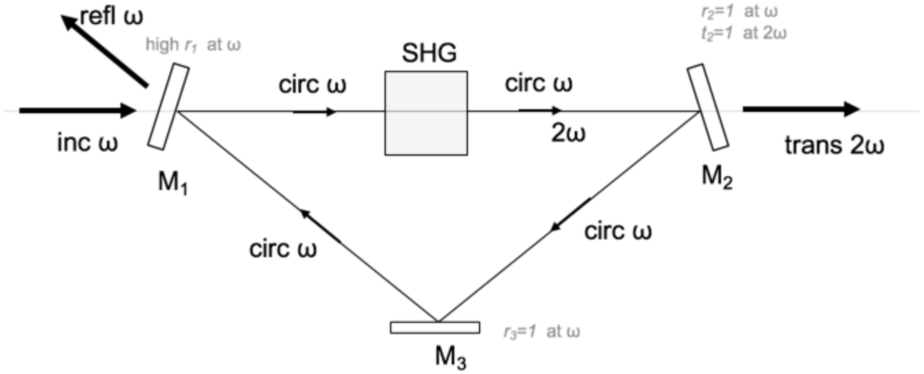
Idea #2



Ideal scenario:

M1 - has high (and optimized) reflection r_1 (in *E*-field) at ω
M2 - reflects ω (~100%) and transmits 2ω (~100%)
M3 - reflects ω (~100%)

Resonating Fundamental



lossless incoupling mirror

$r_1^2 + t_1^2$=1 in *E*-field

$R_1 + T_1 = 1$ in intensity (or power)

α_E=$\alpha_{E,\omega}$

Write self-consistency equation for the *E*- field at ω:

$$\alpha_E E_{circ,\omega} + (1 - r_1) E_{circ,\omega} = t_1 E_{inc,\omega}$$

all other roundtrip losses in the cavity

reflection roundtrip loss

2ω field added due to the pump

→

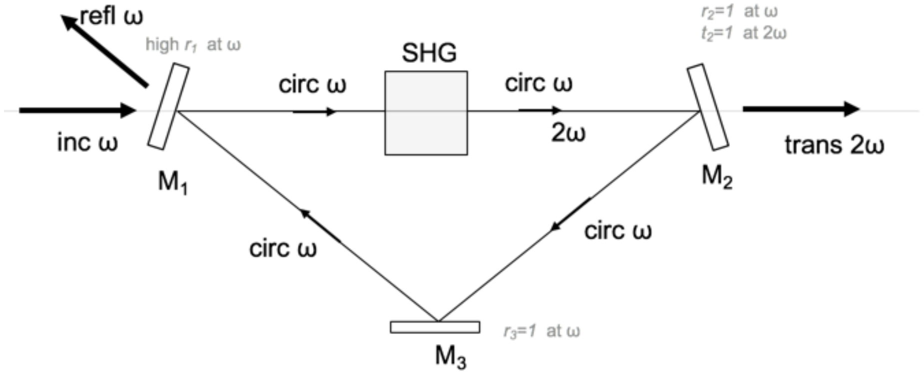
$$E_{circ,\omega} = \frac{t_1 E_{inc,\omega}}{1 - r_1 + \alpha_E} \quad (14.6)$$

circulating *E*-field at ω

circulating ***power*** enhancement at ω

$$\frac{P_{circ,\omega}}{P_{inc,\omega}} = \frac{|E_{circ,\omega}|^2}{|E_{inc,\omega}|^2} = \frac{t_1^2}{(1 - r_1 + \alpha_E)^2} = \frac{1 - r_1^2}{(1 - r_1 + \alpha_E)^2} \approx \frac{2(1 - r_1)}{(1 - r_1 + \alpha_E)^2} \quad (14.7)$$

Resonating Fundamental



Enhancement for circulating ***power*** at ω is maximized when outcoupling loss is equal to other losses: $1 - r_1 = \alpha_E$

'impedance matching' *

$$\frac{P_{circ,\omega}}{P_{inc,\omega}} \approx \frac{2(1-r_1)}{(1-r_1+\alpha_E)^2} \rightarrow \frac{2\alpha_E}{(2\alpha_E)^2} = \frac{1}{2\alpha_E} = \frac{1}{\alpha_I} \qquad (14.8)$$

roundtrip power passive loss in the cavity, e.g. loss in the crystal or diel. mirrors

- can get very high intracavity intensities

$$SHG\ Enh\_factor = |\frac{P_{circ,\omega}}{P_{inc,\omega}}|^2 = \frac{1}{{\alpha_I}^2}$$

However, at high enhancement factors the **power-dependent conversion losses** will be added (which will increase with intracavity power). So the formula will deviate at high enhancement factors.

---

* $T = t^2 = 1 - r^2 \approx 2(1-r) = 2\alpha_E = \alpha_I$

Resonating Fundamental

640 OPTICS LETTERS / Vol. 17, No. 9 / May 1, 1992

85% efficiency for cw frequency doubling from 1.08 to 0.54 μm

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Received December 2, 1991

Conversion efficiency of 85% has been achieved in cw second-harmonic generation from 1.08 to 0.54 μm with a potassium titanyl phosphate crystal inside an external ring cavity. An absolute comparison between the experimental data and a simple theory is made and shows good agreement.

Because of the nonlinear nature of the process and the generally small susceptibilities involved, harmonic generation with high conversion efficiency is most often realized only for pulsed radiation with high intensity for a short period.[1] By contrast, the intensity available for cw operation is usually limited to much smaller values than for pulsed operation, with a concomitant reduction in conversion efficiency. A possible remedy to this circumstance

single KTP crystal inside an external ring cavity of extremely low passive loss. Our measurements are carried out over a range of input fundamental powers up to 700 mW, and a cw conversion efficiency of 85% is achieved. The experimental results are in good absolute agreement with a simple theory, which suggests that efficiencies greater than 90% should be obtainable in our current system once the focusing geometry is optimized.

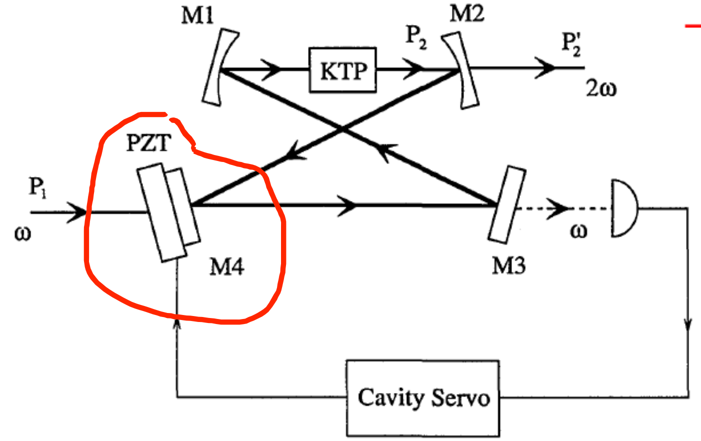

Fig. 1. Schematic of the experimental setup. PZT, piezoelectric transducer.

put infrared light is polarized at 45° with respect to the b or c axis of the crystal. The generated green light is polarized along the b axis. One problem encountered in type II doubling in a resonant cavity is that the indices of refraction are different for the ordinary and extraordinary beams circulating through the KTP crystal, so that longitudinal modes

infrared input power: 700 mW

Resonating Fundamental

September 1, 1994 / Vol. 19, No. 17 / OPTICS LETTERS 1325

82% Efficient continuous-wave frequency doubling of 1.06 μm with a monolithic $MgO:LiNbO_3$ resonator

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Received March 14, 1994

We describe a frequency-doubling monolithic standing-wave resonator made of $MgO:LiNbO_3$ with dielectric mirror coatings for impedance matching near 100 mW input power and near-optimum nonlinear coupling. An external conversion efficiency of 82% has been achieved.

PACS Number: 42.65.Ky

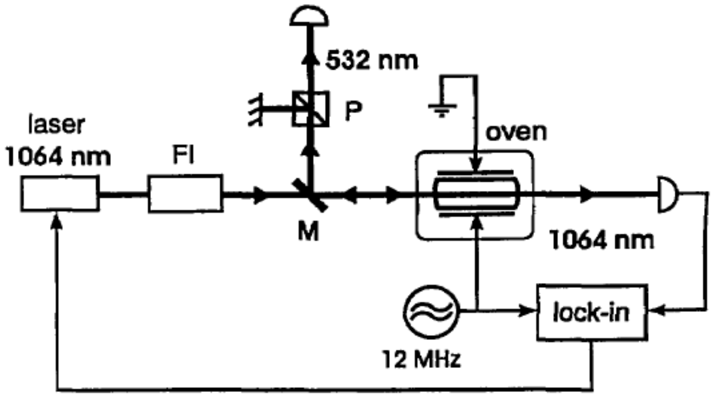

Fig. 3. Experimental setup. The laser beam is mode matched to the frequency doubler mounted in the oven. The Faraday isolator (FI) protects the laser against back-reflected light. M is a dichroic mirror, highly reflective at 532 nm. Polarizer P eliminates residual pump light. The laser frequency is locked to the fundamental wave resonance with the lock-in feedback system.

infrared input power: ~ 100 mW

Yellow SHG inside a laser cavity. Resonating Fundamental

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24 March 2014 | Vol. 22, No. 6 | DOI:10.1364/OE.22.006372 | OPTICS EXPRESS 6372

High-efficiency 20 W yellow VECSEL

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Abstract: A high-efficiency optically pumped vertical-external-cavity surface-emitting laser emitting 20 W at a wavelength around 588 nm is demonstrated. The semiconductor gain chip emitted at a fundamental wavelength around 1170-1180 nm and the laser employed a V-shaped cavity. The yellow spectral range was achieved by intra-cavity frequency doubling using a LBO crystal. The laser could be tuned over a bandwidth of ~26 nm while exhibiting watt-level output powers. The maximum conversion efficiency from absorbed pump power to yellow output was 28% for continuous wave operation. The VECSEL's output could be modulated to generate optical pulses with duration down to 570 ns by directly modulating the pump laser. The high-power pulse operation is a key feature for astrophysics and medical applications while at the same time enables higher slope efficiency than continuous wave operation owing to decreased heating.

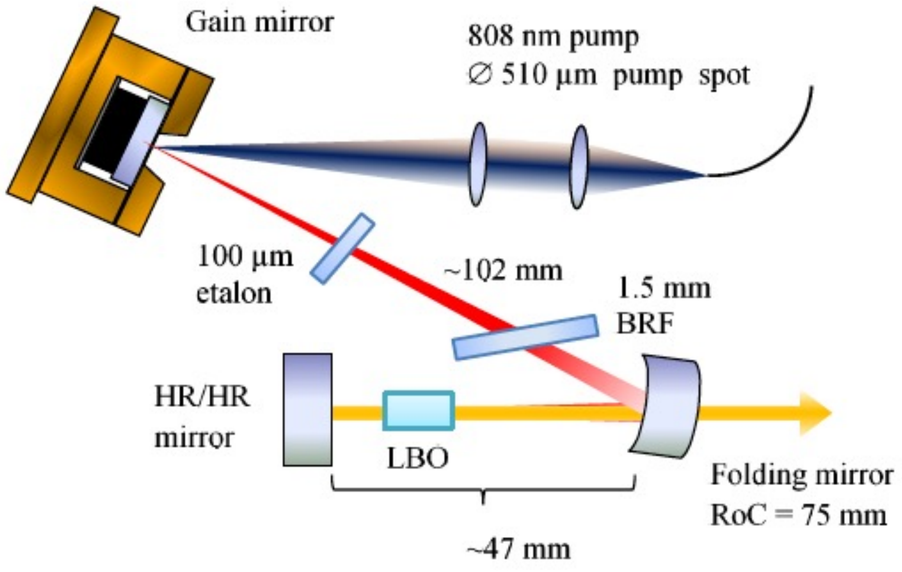


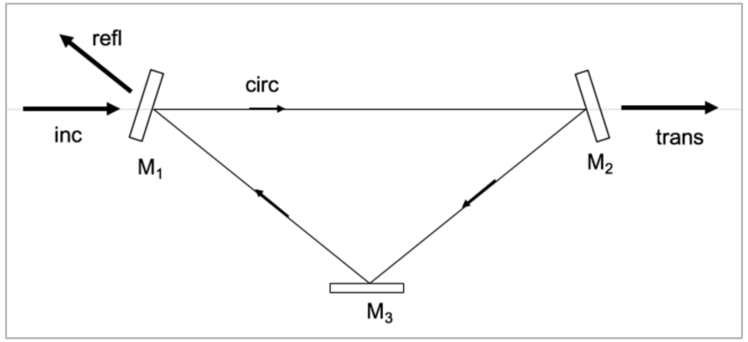
Fig. 2. Schematic illustration of the frequency doubled VECSEL.

Resonant Optical Second Harmonic Generation

What is the catch?

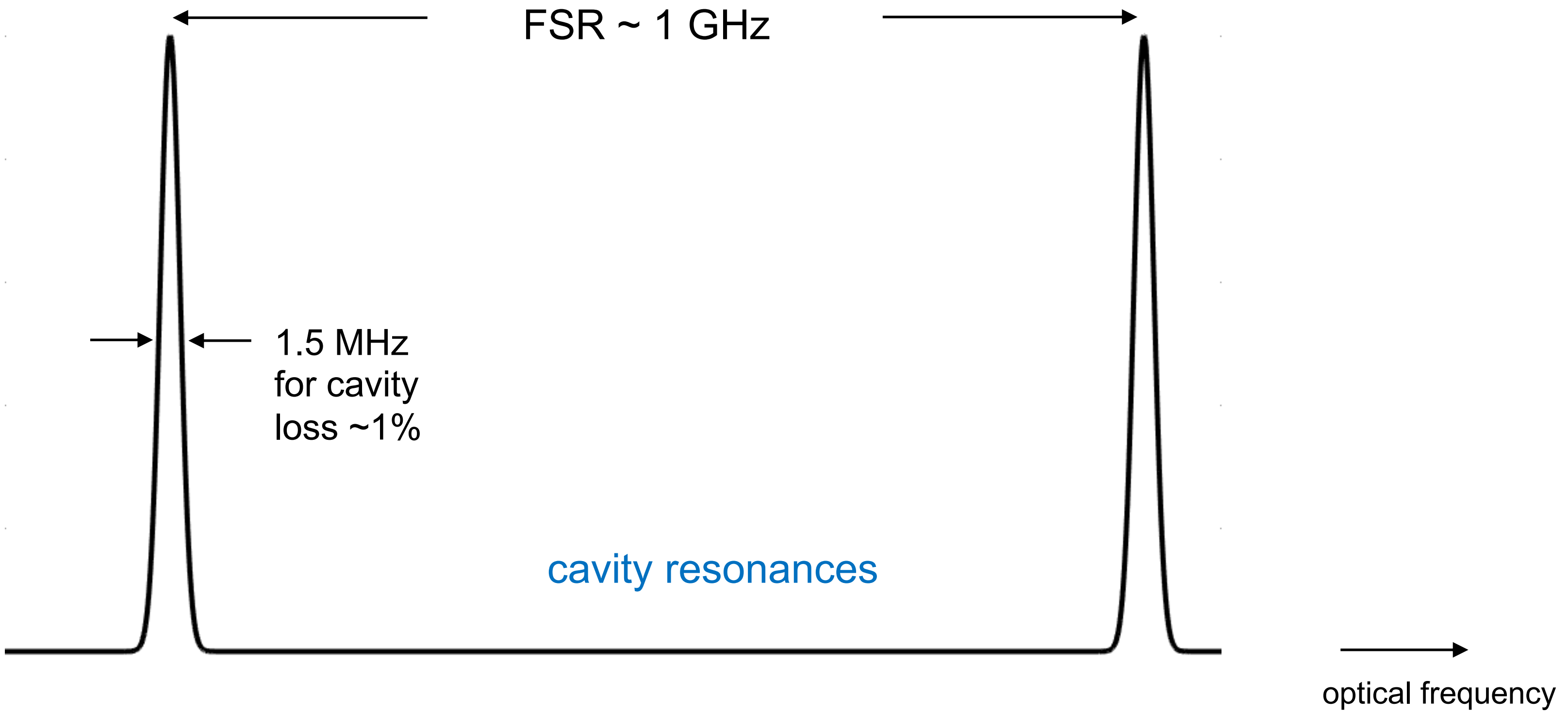
Resonating fundamental wave

Optimum conversion requires **impedance matching** of the resonator, i.e., zero reflected pump power. This is achieved when the input coupler transmission equals the sum of all other losses, including the power-dependent conversion losses.



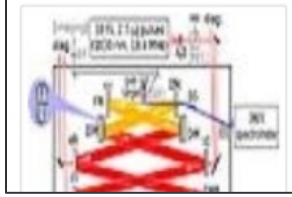
The laser frequency needs to be actively frequency stabilized to the fundamental-wave (or SH-wave) cavity resonance.

cavity length needs to be stable to better than 1-nm accuracy



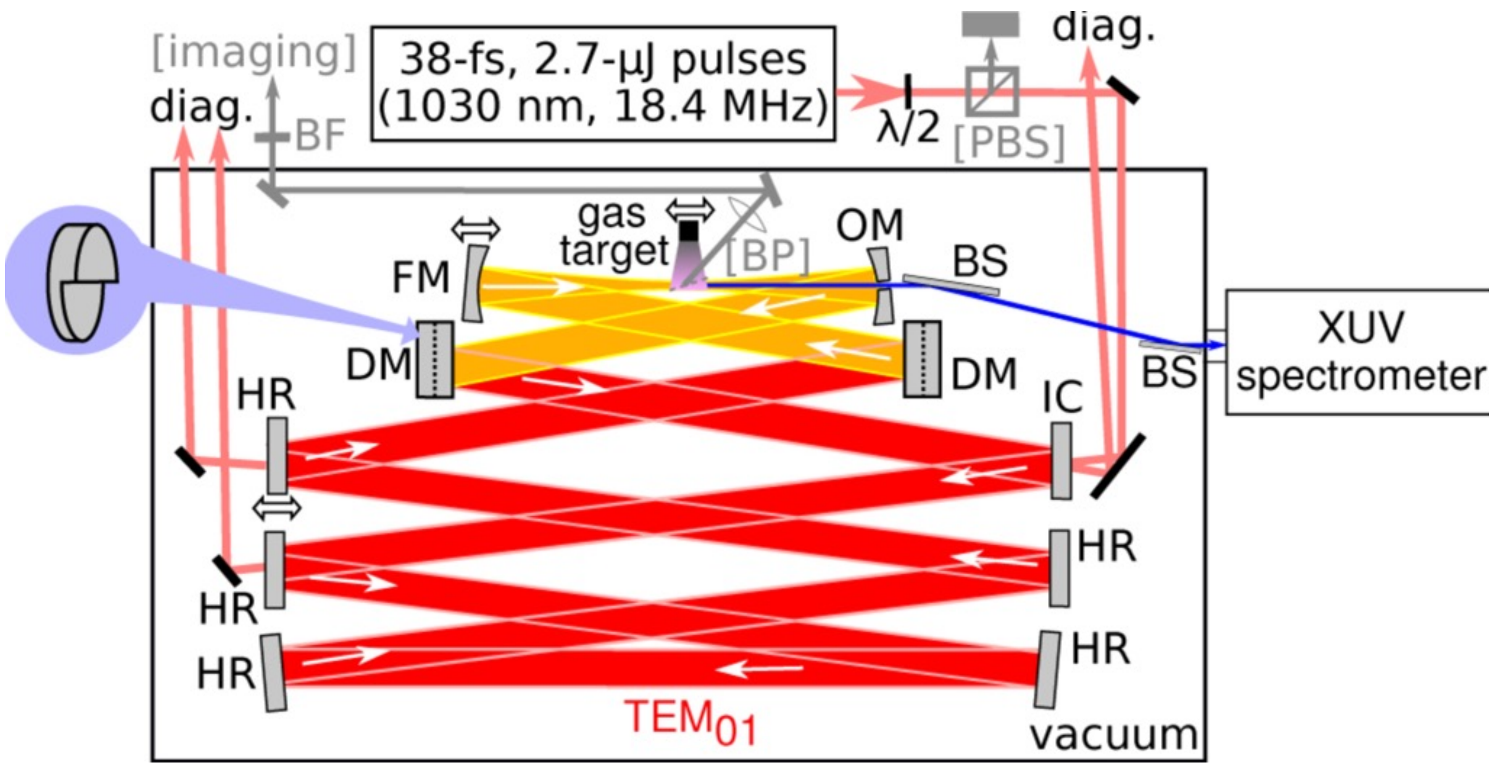
Example from modern physics

Optics Express Vol. 27, Issue 14, pp. 19675-19691 (2019) • https://doi.org/10.1364/OE.27.019675



Cavity-enhanced noncollinear high-harmonic generation

Maximilian Högner, Tobias Saule, Stephan Heinrich, Nikolai Lilienfein, Dominik Esser, Michael Trubetskov, Volodymyr Pervak, and Ioachim Pupeza



Experimental setup. IC: input coupler, HR: highly reflective mirror, DM: HR delay mirror with a stepped surface profile, FM: focusing HR mirror, OM: FM with an on-axis hole for output coupling, BS: IR/XUV beam splitter, diag.: diagnostics. Optional, for imaging the spatial dispersion: BP: Brewster plate, BF: optical bandpass filter, and an attenuator consisting of a half-wave plate ($\lambda/2$) and a polarizing beam splitter (PBS)