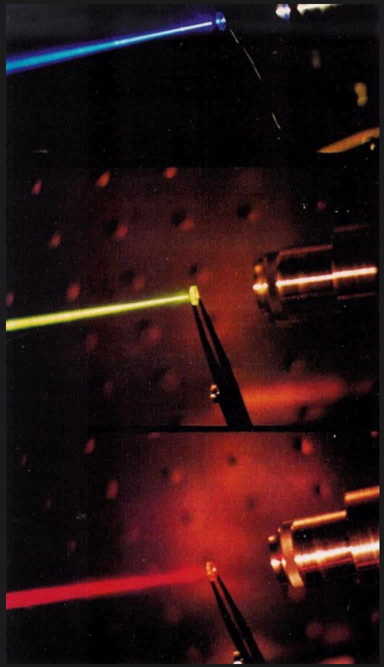
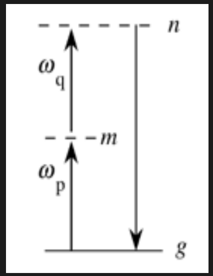
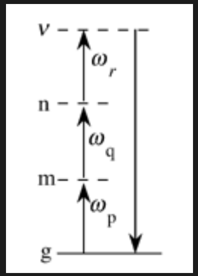
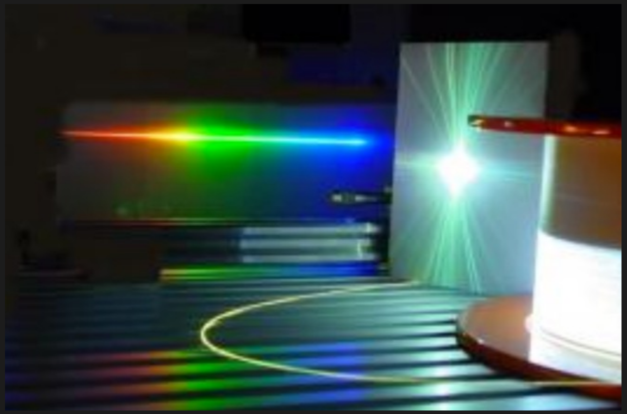
OSE 6334

Nonlinear Optics









Konstantin Vodopyanov

Lecture 1

Introduction to nonlinear optics, logistics,

complex-number algebra for harmonic signals

What this course is about

This course gives a broad introduction to the field of Nonlinear Optics (NLO). The goal is to get you acquainted with the main effects of nonlinear optics and also introduce main equations that govern nonlinear-optical interactions.

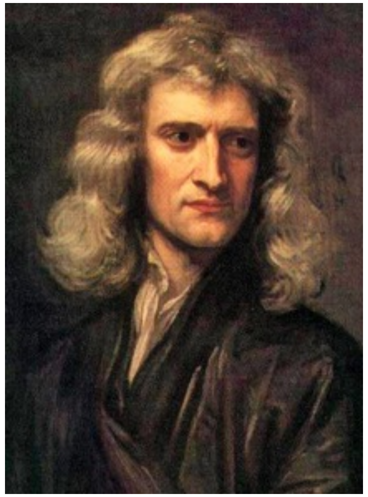
Beginning with a simple electron on a spring model—the course gives comprehensive explanations of second-order and third-order nonlinear effects and their physical origins.

The corse covers nonlinear optics from a combined perspective of:

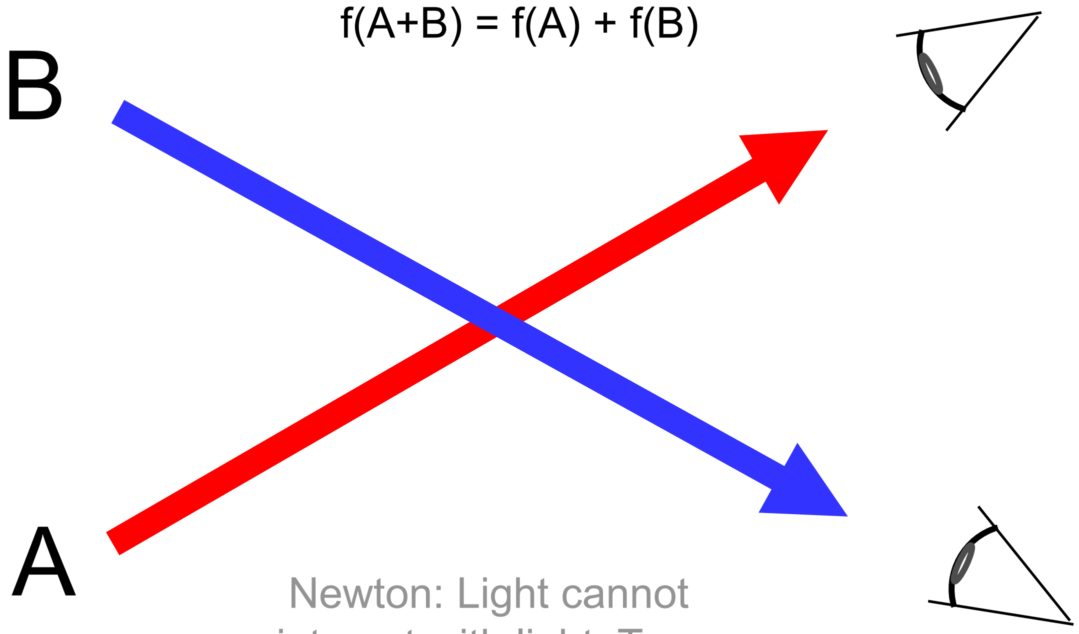
- physics
- optics
- crystallography
- materials science
- and devices

Examples will be given on solving practical problems such as design of a nonlinear frequency converters or a devices using ultrafast lasers.

Newton’s superposition principle



Isaac Newton
1642 –1726

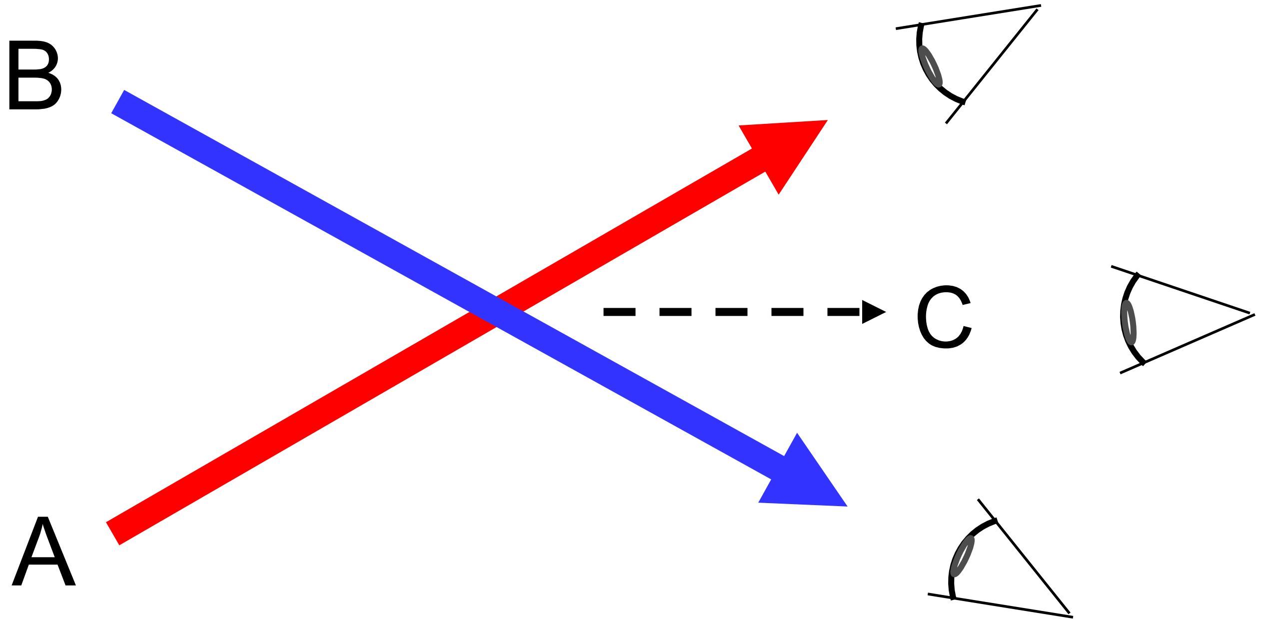


Newton: Light cannot interact with light. Two beams of light can have no effect on each other.

Nonlinear optics (NLO)

Nicolaas Bloembergen
1920–2017
Nobel Prize in Physics (1981)

f(A+B) ≠ f(A) + f(B)
Beams A and B can create beam C



NLO: photons do interact

Linear Optics

- The optical properties such as the refractive index and the absorption coefficient are independent of light intensity.
- The Newton's principle of superposition: light cannot interact with light. Two beams of light in the same region of a linear optical medium can have no effect on each other. Thus light cannot control light.
- The frequency of light cannot be altered by its passage through the medium.

Nonlinear Optics

- The refractive index, and consequently the speed of light in an optical medium does change with the light intensity.
- The absorption coefficient does change with the light intensity.
- The principle of superposition is violated. Light can control light; photons do interact.
- Light can alter its frequency as it passes through a nonlinear optical crystal, e.g. from red to blue and vice versa.

NLO - many fascinating phenomena !

What is the outcome of this course

You will be able to:

Understand modern literature on lasers and their nonlinear optical applications

Distinguish between 2-nd, 3-rd and higher-order nonlinear-optical effects

Do back-of-the-envelope NLO calculations

Explore NLO effects on your own

Syllabus

Home works

- One home work per week (~ 3 problems)
- Assignment give on: Thursday
- Home work (submit via Webcourses) due: next Wednesday before midnigh

Final exam

Apr 27 2023, Thursday, 3:00–6:00 pm

In the form of 10-min students' presentation (~ 10 slides)

Sources:

- Articles from different journals: Scientific American, Nature, Science, Optica, Optics Letters, New Focus, New York Times, etc ....
- Can also be an original experimental or theoretical project of your choice

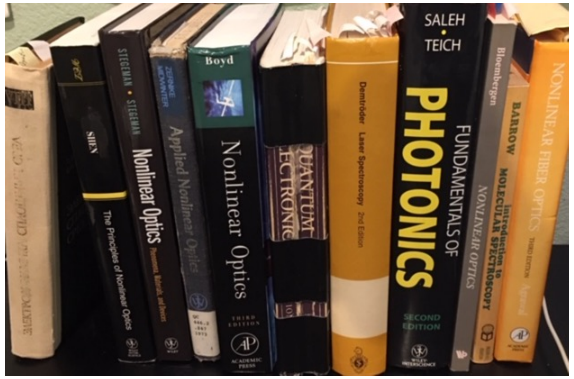
Texbooks

(there is no 'required' textbook for this course)

1. R. W. Boyd, *Nonlinear Optics*, 3rd Edition, (Academic Press, 2008).
2. G. I. Stegeman, R. A. Stegeman, *Nonlinear Optics: Phenomena, Materials and Devices*, (Wiley 2012)

} These are two main textbooks. Boyd is easier to read than Stegeman, although Boyd has some unconventional definitions; Stegeman has more formalism, but gives nice physical models for different NLO effects

3. Y. R. Shen, *The Principles of Nonlinear Optics* (Wiley 2003) – nice book, but heavier formalism than in other books
4. B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics* (Wiley 2007) NLO chapter – very clearly written
5. A. Yariv, *Quantum Electronics* (Wiley 1989) – nice chapters on quantum mechanics for NLO



books that I used to prepare this course

Lecture notes

The lecture notes will be placed on Webcourses' "Files" as the course progresses.

For example, notes for the next lecture will be placed no later than the night before.

You can interrupt me at any time during a lecture if you have questions

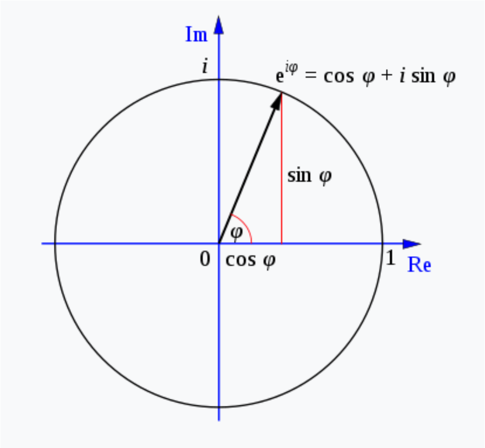
Complex numbers

Euler's formula

$$e^{ix} = cosx + i\ sinx$$

$$e^{i\pi} = -1$$

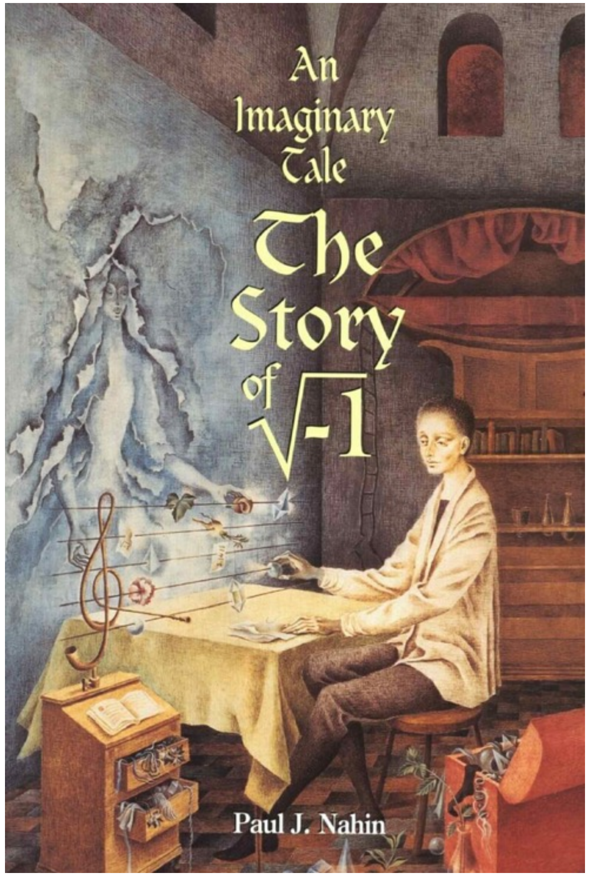
x can be real or complex number!



conversely:

$$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$$



Complex numbers

$$e^i \; =?$$

$$i^e \; =?$$

$$\cos(i) \; =?$$

$$\frac{\ln(-1)}{i} \; =?$$

Complex numbers

$$e^{i} = ?$$

$$= \cos(1) + i\sin(1) = 0.5403 + 0.8415i$$

$$i^{e} = ?$$

$$= \left(e^{i\frac{\pi}{2}}\right)^{e} = e^{i\frac{\pi e}{2}} = -0.4282 - 0.9037i$$

$$\cos(i) = ?$$

$$= \frac{1}{2}\left(e^{-1} + e^{1}\right) = 1.54$$

$$\frac{\ln(-1)}{i} = ?$$

$$= \frac{\ln(e^{i\pi})}{i} = \frac{i\pi}{i} = \pi$$

Complex numbers

Calculations with harmonic waves are simplified by the use of **exponentials** vs trigonometric functions, for example:

Electrical field strength of the optical field: **E**(t) = $E\cos(\omega t+\varphi)$ – all values are real here

->

E(t) = $Real\ \{ E e^{i(\omega t+\varphi)} \}$,

or even better:

->

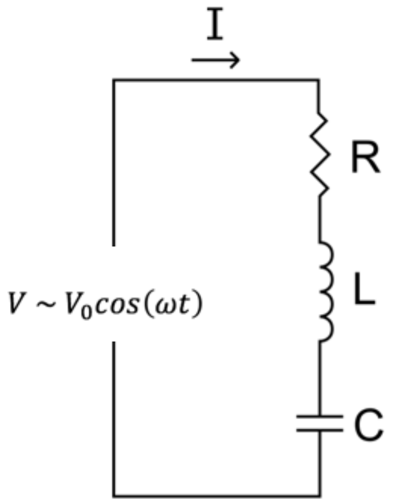
E(t) = $Real\ \{\tilde{E} e^{i\omega t}\}$, with $\tilde{E}$ being a complex amplitide $\tilde{E} = E e^{i\varphi}$

If the operations on the field are linear, one may drop the *Real* symbol and operate directly with the complex function $\tilde{E} e^{i\omega t}$.

The *Real* part of the final expression is used to represent the **physical quantity** in question.

Complex numbers

Example: L-C-R electric circuits



$$RI + L\frac{d}{dt}I + \frac{1}{C}\int I dt = \mathrm{V(t)};$$

$$\int I dt = q; \;\; I = dq/dt$$

$$\frac{d^2}{dt^2}I(t) + \frac{R}{L}\frac{d}{dt}I(t) + \frac{1}{LC}I(t) = \frac{1}{L}\frac{d}{dt}V(t)$$

$$V(t) = V_0 cos(\omega t) = Real(V_0 e^{i\omega t}) \quad \rightarrow V_0 e^{i\omega t}$$

look for a solution in the form: $I(t) = \tilde{I} e^{i\omega t}$

complex

and get immediately $$\tilde{I} = \frac{V_0}{\underbrace{R + i\omega L - i/\omega C}_{iZ}} = \frac{V_0}{R+iZ} = \frac{V_0 e^{i\varphi}}{\sqrt{R^2+Z^2}} = I_0 e^{i\varphi}, \qquad I_0 = \frac{V_0}{\sqrt{R^2+Z^2}}$$

$$\varphi = -\mathrm{atan}(Z/R)$$

the physical reality: $I(t) = Re(\tilde{I} e^{i\omega t}) = I_0 cos(\omega t + \varphi)$

$e^{i\varphi}$ simply means phase shift

Complex numbers

However, when dealing with expressions which involve **nonlinear operations such as squaring**, one must take the real parts first and operate with these alone.

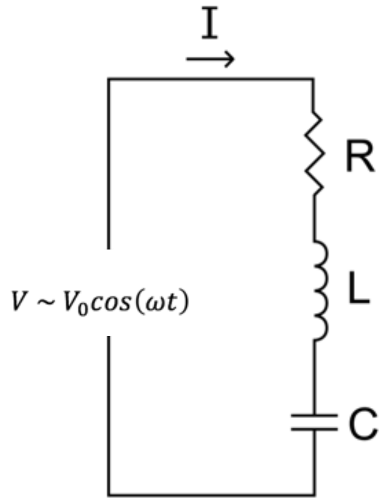
Need to expand:

$$E(t) = Real\left(Ee^{i\omega t}\right) = \frac{1}{2}\left(Ee^{i\omega t} + E^{*}e^{-i\omega t}\right) = \frac{1}{2}\left(Ee^{i\omega t} + c.c.\right)$$

complex conjugate

Complex numbers

what is the <u>average</u> power dissipated ?



nonlinear expression

$$\bar{P} = \overline{V \times I} = \overline{\frac{1}{2}(V_0 e^{i\omega t} + V_0 e^{-i\omega t}) \times \frac{1}{2}(\tilde{I} e^{i\omega t} + \tilde{I}^* e^{-i\omega t})}$$

$$= \overline{\frac{1}{2}(V_0 e^{i\omega t} + V_0 e^{-i\omega t}) \times \frac{1}{2}(I_0 e^{i(\omega t+\varphi)} + I_0 e^{-i(\omega t+\varphi)})}$$

$$= \overline{\frac{1}{4} V_0 I_0 (e^{i\omega t} e^{i(\omega t+\varphi)} + e^{i\varphi} + e^{-i\omega t} e^{-i(\omega t+\varphi)} + e^{-i\varphi})}$$

average to zero

$$= \overline{\frac{1}{4} V_0 I_0 (e^{i\varphi} + e^{-i\varphi})} \quad = \frac{1}{2} V_0 I_0 cos(\varphi)$$

makes sense!

incorrect way: $\bar{P} = Re\{\overline{V_0 e^{i\omega t} \times I_0 e^{i(\omega t+\varphi)}}\} = Re\{\overline{V_0 \times I_0 e^{i(2\omega t+\varphi)}}\} = 0$

Complex numbers

In any <u>linear</u> equation of the form: $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = F cos(\omega t)$

- can use exponential form for the solution: $x = xe^{i\omega t}$ x -complex

and get immediately: $$x = \frac{Fe^{i\omega t}}{-\omega^2 + i\gamma\omega + \omega_0^2} = \frac{Fe^{i\omega t}}{(\omega_0^2 - \omega^2) + i\gamma\omega} =$$

$$= \frac{Fe^{i\omega t + i\varphi}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}}, \qquad \varphi = -\text{atan}(\gamma\omega/(\omega_0^2 - \omega^2))$$

the physical reality: $x \rightarrow Re(x) = \frac{F}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}} cos(\omega t + \varphi)$

Complex numbers

An alternative without using an exponent (real numbers only)
Again, find the solution of the equation:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = Fcos(\omega t)$$

in the form: $x = A\ cos(\omega t) + B\ sin(\omega t)$

we get :

$$-\omega^2 A\ cos(\omega t) - \omega^2 B\ sin(\omega t) - \omega A\ sin(\omega t) + \omega B\ cos(\omega t) + \omega^2 (A\ cos(\omega t) + B\ sin(\omega t)) = Fcos(\omega t)$$

By separating *sine* and *cosine* parts, get two equaions and find A and B, and eventualy get the same resut as before ...

but in a much more complicated way

Complex numbers, final comment

Rationale for using an exponent instead of sin & cos

Imagine you have this linear diff. equation

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = Fcos(\omega t)$$

Look for solution in the form:

$x = A\ cos(\omega t) + B\ sin(\omega t)$, same as: $x = xcos(\omega t + \varphi)$

$$x = xcos(\omega t + \varphi) = \frac{1}{2}\{xe^{i\omega t + i\varphi} + xe^{-i\omega t - i\varphi}\}$$

$Fcos(\omega t)$ -> $\frac{1}{2}\{Fe^{i\omega t} + Fe^{-i\omega t}\}$

Plug those into main equation and separate terms with $i\omega t$ and $-i\omega t$

$$-\omega^2 xe^{i\varphi} + i\omega\gamma xe^{i\varphi} + \omega_0^2 xe^{i\varphi} = F$$
$$-\omega^2 xe^{-i\varphi} - i\omega\gamma xe^{-i\varphi} + \omega_0^2 xe^{-i\varphi} = F$$

The 2nd equation is the conjugate of the 1st equation. Hence it is not needed!

Do the **quiz** today or tomorrow so that you will be registered for the course.